CONSUMABLE WORKBOOKS  Many of the worksheets contained in the Chapter Resource Masters booklets are available as consumable workbooks in both English and Spanish.

<table>
<thead>
<tr>
<th></th>
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<tbody>
<tr>
<td>Study Guide and Intervention Workbook</td>
<td>0-07-890848-5</td>
<td>978-0-07-890848-4</td>
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<tr>
<td>Homework Practice Workbook</td>
<td>0-07-890849-3</td>
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**Spanish Version**

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<tr>
<td>Homework Practice Workbook</td>
<td>0-07-890853-1</td>
<td>978-0-07-890853-8</td>
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</table>

ANSWERS FOR WORKBOOKS  The answers for Chapter 8 of these workbooks can be found in the back of this Chapter Resource Masters booklet.

**StudentWorks Plus™**  This CD-ROM includes the entire Student Edition text along with the English workbooks listed above.

**TeacherWorks Plus™**  All of the materials found in this booklet are included for viewing, printing, and editing in this CD-ROM.

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Teacher’s Guide to Using the

Chapter 8 Resource Masters

The Chapter 8 Resource Masters includes the core materials needed for Chapter 8. These materials include worksheets, extensions, and assessment options. The answers for these pages appear at the back of this booklet.

All of the materials found in this booklet are included for viewing and printing on the TeacherWorks Plus™ CD-ROM.

Chapter Resources

Student-Built Glossary (pages 1–2) These masters are a student study tool that presents up to twenty of the key vocabulary terms from the chapter. Students are to record definitions and/or examples for each term. You may suggest that students highlight or star the terms with which they are not familiar. Give this to students before beginning Lesson 8-1. Encourage them to add these pages to their mathematics study notebooks. Remind them to complete the appropriate words as they study each lesson.

Anticipation Guide (pages 3–4) This master, presented in both English and Spanish, is a survey used before beginning the chapter to pinpoint what students may or may not know about the concepts in the chapter. Students will revisit this survey after they complete the chapter to see if their perceptions have changed.

Lesson Resources

Study Guide and Intervention These masters provide vocabulary, key concepts, additional worked-out examples and Check Your Progress exercises to use as a reteaching activity. It can also be used in conjunction with the Student Edition as an instructional tool for students who have been absent.

Skills Practice This master focuses more on the computational nature of the lesson. Use as an additional practice option or as homework for second-day teaching of the lesson.

Practice This master closely follows the types of problems found in the Exercises section of the Student Edition and includes word problems. Use as an additional practice option or as homework for second-day teaching of the lesson.

Word Problem Practice This master includes additional practice in solving word problems that apply the concepts of the lesson. Use as an additional practice or as homework for second-day teaching of the lesson.

Enrichment These activities may extend the concepts of the lesson, offer an historical or multicultural look at the concepts, or widen students’ perspectives on the mathematics they are learning. They are written for use with all levels of students.

Graphing Calculator or Spreadsheet Activities

These activities present ways in which technology can be used with the concepts in some lessons of this chapter. Use as an alternative approach to some concepts or as an integral part of your lesson presentation.
Assessment Options
The assessment masters in the Chapter 8 Resource Masters offer a wide range of assessment tools for formative (monitoring) assessment and summative (final) assessment.

Student Recording Sheet  This master corresponds with the standardized test practice at the end of the chapter.

Extended–Response Rubric  This master provides information for teachers and students on how to assess performance on open-ended questions.

Quizzes  Four free-response quizzes offer assessment at appropriate intervals in the chapter.

Mid-Chapter Test  This 1-page test provides an option to assess the first half of the chapter. It parallels the timing of the Mid-Chapter Quiz in the Student Edition and includes both multiple-choice and free-response questions.

Vocabulary Test  This test is suitable for all students. It includes a list of vocabulary words and 10 questions to assess students’ knowledge of those words. This can also be used in conjunction with one of the leveled chapter tests.

Leveled Chapter Tests
• Form 1 contains multiple-choice questions and is intended for use with below grade level students.
• Forms 2A and 2B contain multiple-choice questions aimed at on grade level students. These tests are similar in format to offer comparable testing situations.
• Forms 2C and 2D contain free-response questions aimed at on grade level students. These tests are similar in format to offer comparable testing situations.
• Form 3 is a free-response test for use with above grade level students.

All of the above mentioned tests include a free-response Bonus question.

Extended-Response Test  Performance assessment tasks are suitable for all students. Sample answers and a scoring rubric are included for evaluation.

Standardized Test Practice  These three pages are cumulative in nature. It includes three parts: multiple-choice questions with bubble-in answer format, griddable questions with answer grids, and short-answer free-response questions.

Answers
• The answers for the Anticipation Guide and Lesson Resources are provided as reduced pages.
• Full-size answer keys are provided for the assessment masters.
This is an alphabetical list of the key vocabulary terms you will learn in Chapter 8. As you study the chapter, complete each term’s definition or description. Remember to add the page number where you found the term. Add these pages to your Geometry Study Notebook to review vocabulary at the end of the chapter.

<table>
<thead>
<tr>
<th>Vocabulary Term</th>
<th>Found on Page</th>
<th>Definition/Description/Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>angle of depression</td>
<td></td>
<td></td>
</tr>
<tr>
<td>angle of elevation</td>
<td></td>
<td></td>
</tr>
<tr>
<td>component form</td>
<td></td>
<td></td>
</tr>
<tr>
<td>cosine</td>
<td></td>
<td></td>
</tr>
<tr>
<td>geometric mean</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Law of Cosines</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Law of Sines</td>
<td></td>
<td></td>
</tr>
<tr>
<td>magnitude</td>
<td></td>
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</table>

(continued on the next page)
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<tr>
<th>Vocabulary Term</th>
<th>Found on Page</th>
<th>Definition/Description/Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pythagorean triple</td>
<td></td>
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<tr>
<td>resultant</td>
<td></td>
<td></td>
</tr>
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<td>sine</td>
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<td></td>
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<td>tangent</td>
<td></td>
<td></td>
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<td>trigonometric ratio</td>
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<tr>
<td>trigonometry</td>
<td></td>
<td></td>
</tr>
<tr>
<td>vector</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
## Anticipation Guide

### Right Triangles and Trigonometry

#### Step 1

**Before you begin Chapter 8**

- Read each statement.
- Decide whether you Agree (A) or Disagree (D) with the statement.
- Write A or D in the first column OR if you are not sure whether you agree or disagree, write NS (Not Sure).

<table>
<thead>
<tr>
<th>STEP 1 A, D, or NS</th>
<th>Statement</th>
<th>STEP 2 A or D</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>The geometric mean between two numbers is the positive square root of their product.</td>
<td></td>
</tr>
<tr>
<td>2.</td>
<td>An altitude drawn from the right angle of a right triangle to its hypotenuse separates the triangle into two congruent triangles.</td>
<td></td>
</tr>
<tr>
<td>3.</td>
<td>In a right triangle, the length of the hypotenuse is equal to the sum of the lengths of the legs.</td>
<td></td>
</tr>
<tr>
<td>4.</td>
<td>If any triangle has sides with lengths 3, 4, and 5, then that triangle is a right triangle.</td>
<td></td>
</tr>
<tr>
<td>5.</td>
<td>If the two acute angles of a right triangle are 45°, then the length of the hypotenuse is $\sqrt{2}$ times the length of either leg.</td>
<td></td>
</tr>
<tr>
<td>6.</td>
<td>In any triangle whose angle measures are 30°, 60°, and 90°, the hypotenuse is $\sqrt{3}$ times as long as the shorter leg.</td>
<td></td>
</tr>
<tr>
<td>7.</td>
<td>The sine ratio of an angle of a right triangle is equal to the length of the adjacent side divided by the length of the hypotenuse.</td>
<td></td>
</tr>
<tr>
<td>8.</td>
<td>The tangent of an angle of a right triangle whose sides have lengths 3, 4, and 5 will be smaller than the tangent of an angle of a right triangle whose sides have lengths 6, 8, and 10.</td>
<td></td>
</tr>
<tr>
<td>9.</td>
<td>Trigonometric ratios can be used to solve problems involving angles of elevation and angles of depression.</td>
<td></td>
</tr>
<tr>
<td>10.</td>
<td>The Law of Sines can only be used in right triangles.</td>
<td></td>
</tr>
</tbody>
</table>

#### Step 2

**After you complete Chapter 8**

- Reread each statement and complete the last column by entering an A or a D.
- Did any of your opinions about the statements change from the first column?
- For those statements that you mark with a D, use a piece of paper to write an example of why you disagree.
## Ejercicios preparatorios

### Triángulos rectángulos y trigonometría

#### Paso 1

**Antes de comenzar el Capítulo 8**

- Lee cada enunciado.
- Decide si estás de acuerdo (A) o en desacuerdo (D) con el enunciado.
- Escribe A o D en la primera columna O si no estás seguro(a) de la respuesta, escribe NS (No estoy seguro(a).

<table>
<thead>
<tr>
<th>PASO 1 A, D o NS</th>
<th>Enunciado</th>
<th>PASO 2 A o D</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>La media geométrica de dos números es la raíz cuadrada positiva de su producto.</td>
<td></td>
</tr>
<tr>
<td>2.</td>
<td>Una altitud que se dibuja desde el ángulo recto de un triángulo rectángulo hasta su hipotenusa divide el triángulo en dos triángulos congruentes.</td>
<td></td>
</tr>
<tr>
<td>3.</td>
<td>En un triángulo rectángulo, la longitud de la hipotenusa es igual a la suma de las longitudes de los catetos.</td>
<td></td>
</tr>
<tr>
<td>4.</td>
<td>Si un triángulo rectángulo tiene lados con longitudes 3, 4 y 5, entonces cualquier triángulo es un triángulo rectángulo.</td>
<td></td>
</tr>
<tr>
<td>5.</td>
<td>Si los dos ángulos agudos de un triángulo rectángulo son de 45°, entonces la longitud de la hipotenusa es ( \sqrt{2} ) veces la longitud de cualquiera de los catetos.</td>
<td></td>
</tr>
<tr>
<td>6.</td>
<td>En cualquiera triángulo cuyas medidas de ángulos sean de 30°, 60° y 90°, la hipotenusa es ( \sqrt{3} ) veces tan larga como el cateto más corto.</td>
<td></td>
</tr>
<tr>
<td>7.</td>
<td>La razón del seno para un ángulo en un triángulo rectángulo es igual a la longitud del lado adyacente dividido entre la longitud de la hipotenusa.</td>
<td></td>
</tr>
<tr>
<td>8.</td>
<td>La tangente para un ángulo de un triángulo rectángulo cuyos lados tienen longitudes 3, 4 y 5 será menor que la tangente de un ángulo en un triángulo rectángulo cuyos lados tienen longitudes 6, 8 y 10.</td>
<td></td>
</tr>
<tr>
<td>9.</td>
<td>Las razones trigonométricas se pueden usar para resolver problemas de ángulos de elevación y ángulos de depresión.</td>
<td></td>
</tr>
<tr>
<td>10.</td>
<td>La ley de los senos sólo se puede usar con triángulos rectángulos.</td>
<td></td>
</tr>
</tbody>
</table>

#### Paso 2

**Después de completar el Capítulo 8**

- Vuelve a leer cada enunciado y completa la última columna con una A o una D.
- ¿Cambió cualquiera de tus opiniones sobre los enunciados de la primera columna?
- En una hoja de papel aparte, escribe un ejemplo de por qué estás en desacuerdo con los enunciados que marcaste con una D.
Geometric Mean

The geometric mean between two numbers is the positive square root of their product. For two positive numbers $a$ and $b$, the geometric mean of $a$ and $b$ is the positive number $x$ in the proportion $\frac{a}{x} = \frac{x}{b}$. Cross multiplying gives $x^2 = ab$, so $x = \sqrt{ab}$.

**Example**

Find the geometric mean between each pair of numbers.

a. 12 and 3

\[
x = \sqrt{ab} \quad \text{Definition of geometric mean}
\]
\[
= \sqrt{12 \cdot 3} \quad a = 12 \text{ and } b = 3
\]
\[
= \sqrt{2 \cdot 2 \cdot 3} \cdot 3 \quad \text{Factor.}
\]
\[
= 6 \quad \text{Simplify.}
\]

The geometric mean between 12 and 3 is 6.

b. 8 and 4

\[
x = \sqrt{ab} \quad \text{Definition of geometric mean}
\]
\[
= \sqrt{8 \cdot 4} \quad a = 8 \text{ and } b = 4
\]
\[
= \sqrt{(2 \cdot 4) \cdot 4} \quad \text{Factor.}
\]
\[
= \sqrt{16 \cdot 2} \quad \text{Associative Property}
\]
\[
= 4 \sqrt{2} \quad \text{Simplify.}
\]

The geometric mean between 8 and 4 is $4 \sqrt{2}$ or about 5.7.

**Exercises**

Find the geometric mean between each pair of numbers.

1. 4 and 4
2. 4 and 6
3. 6 and 9
4. $\frac{1}{2}$ and 2
5. 12 and 20
6. 4 and 25
7. 16 and 30
8. 10 and 100
9. $\frac{1}{2}$ and $\frac{1}{4}$
10. 17 and 3
11. 4 and 16
12. 3 and 24
Geometric Mean

Geometric Means in Right Triangles In the diagram, \( \triangle ABC \sim \triangle ADB \sim \triangle BDC \). An altitude to the hypotenuse of a right triangle forms two right triangles. The two triangles are similar and each is similar to the original triangle.

**Example 1** Use right \( \triangle ABC \) with \( BD \perp AC \). Describe two geometric means.

a. \( \triangle ADB \sim \triangle BDC \) so \( \frac{AD}{BD} = \frac{CD}{BD} \).

In \( \triangle ABC \), the altitude is the geometric mean between the two segments of the hypotenuse.

b. \( \triangle ABC \sim \triangle ADB \) and \( \triangle ABC \sim \triangle BDC \), so \( \frac{AC}{AB} = \frac{AD}{BD} \) and \( \frac{AC}{BC} = \frac{BC}{DC} \).

In \( \triangle ABC \), each leg is the geometric mean between the hypotenuse and the segment of the hypotenuse adjacent to that leg.

**Example 2** Find \( x, y, \) and \( z \).

\[
15 = \sqrt{RP \cdot SP} \quad \text{Geometric Mean (Leg) Theorem}
\]

\[
15 = \sqrt{25x}
\]

\[
225 = 25x
\]

\[
x = 9
\]

Then

\[
y = RP - SP
\]

\[
y = 25 - 9
\]

\[
y = 16
\]

\[
z = \sqrt{RS \cdot RP}
\]

\[
z = \sqrt{16 \cdot 25}
\]

\[
z = \sqrt{400}
\]

\[
z = 20
\]

Exercises

Find \( x, y, \) and \( z \) to the nearest tenth.

1. \[
\begin{array}{c}
1 \\
3 \\
x
\end{array}
\]

2. \[
\begin{array}{c}
2 \\
5 \\
x
\end{array}
\]

3. \[
\begin{array}{c}
1 \\
8 \\
x
\end{array}
\]

4. \[
\begin{array}{c}
\sqrt{12} \\
y \\
\sqrt{3} \\
1
\end{array}
\]

5. \[
\begin{array}{c}
z \\
2 \\
x
\end{array}
\]

6. \[
\begin{array}{c}
x \\
y \\
z
\end{array}
\]
8-1 Skills Practice

Geometric Mean

Find the geometric mean between each pair of numbers.

1. 2 and 8
2. 9 and 36
3. 4 and 7
4. 5 and 10
5. 28 and 14
6. 7 and 36

Write a similarity statement identifying the three similar triangles in the figure.

7. \( \triangle ADC \) and \( \triangle BDC \)
8. \( \triangle LMP \) and \( \triangle LNP \)

9. \( \triangle EGH \) and \( \triangle FGH \)
10. \( \triangle SRT \) and \( \triangle SUT \)

Find \( x, y \) and \( z \).

11. \( \triangle XYZ \) with \( x = 3, y = 9 \)
12. \( \triangle XYZ \) with \( x = 10, y = 4 \)
13. \( \triangle XYZ \) with \( x = 15, y = 4 \)
14. \( \triangle XYZ \) with \( x = 5, y = 2 \)
8-1 Practice

Geometric Mean

Find the geometric mean between each pair of numbers.

1. 8 and 12
2. 3 and 15
3. \( \frac{4}{5} \) and 2

Write a similarity statement identifying the three similar triangles in the figure.

4.

Write a similarity statement identifying the three similar triangles in the figure.

5.

Find \( x \), \( y \), and \( z \).

6.

7.

8.

9.

10. CIVIL An airport, a factory, and a shopping center are at the vertices of a right triangle formed by three highways. The airport and factory are 6.0 miles apart. Their distances from the shopping center are 3.6 miles and 4.8 miles, respectively. A service road will be constructed from the shopping center to the highway that connects the airport and factory. What is the shortest possible length for the service road? Round to the nearest hundredth.
1. **SQUARES** Wilma has a rectangle of dimensions $\ell$ by $w$. She would like to replace it with a square that has the same area. What is the side length of the square with the same area as Wilma’s rectangle?

2. **EQUALITY** Gretchen computed the geometric mean of two numbers. One of the numbers was 7 and the geometric mean turned out to be 7 as well. What was the other number?

3. **VIEWING ANGLE** A photographer wants to take a picture of a beach front. His camera has a viewing angle of 90° and he wants to make sure two palm trees located at points $A$ and $B$ in the figure are just inside the edges of the photograph.

He walks out on a walkway that goes over the ocean to get the shot. If his camera has a viewing angle of 90°, at what distance down the walkway should he stop to take his photograph?

4. **EXHIBITIONS** A museum has a famous statue on display. The curator places the statue in the corner of a rectangular room and builds a 15-foot-long railing in front of the statue. Use the information below to find how close visitors will be able to get to the statue.

5. **CLIFFS** A bridge connects to a tunnel as shown in the figure. The bridge is 180 feet above the ground. At a distance of 235 feet along the bridge out of the tunnel, the angle to the base and summit of the cliff is a right angle.

   a. What is the height of the cliff? Round to the nearest whole number.

   b. How high is the cliff from base to summit? Round to the nearest whole number.

   c. What is the value of $d$? Round to the nearest whole number.
Mathematics and Music

Pythagoras, a Greek philosopher who lived during the sixth century B.C., believed that all nature, beauty, and harmony could be expressed by whole-number relationships. Most people remember Pythagoras for his teachings about right triangles. (The sum of the squares of the legs equals the square of the hypotenuse.) But Pythagoras also discovered relationships between the musical notes of a scale. These relationships can be expressed as ratios.

<table>
<thead>
<tr>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>A</th>
<th>B</th>
<th>C’</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>8</td>
<td>4</td>
<td>3</td>
<td>2</td>
<td>3</td>
<td>8</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>9</td>
<td>5</td>
<td>4</td>
<td>3</td>
<td>5</td>
<td>15</td>
<td>2</td>
</tr>
</tbody>
</table>

When you play a stringed instrument, you produce different notes by placing your finger on different places on a string. This is the result of changing the length of the vibrating part of the string.

The C string can be used to produce F by placing a finger $\frac{3}{4}$ of the way along the string.

Suppose a C string has a length of 16 inches. Write and solve proportions to determine what length of string would have to vibrate to produce the remaining notes of the scale.

1. D
2. E
3. F
4. G
5. A
6. B
7. C’

8. Complete to show the distance between finger positions on the 16-inch C string for each note. For example, C(16) – D(14 $\frac{2}{9}$) = 1 $\frac{7}{9}$.

C $\frac{17}{9}$ in. D _______ E _______ F _______ G _______ A _______ B _______ C’
The Pythagorean Theorem

In a right triangle, the sum of the squares of the lengths of the legs equals the square of the length of the hypotenuse. If the three whole numbers \(a\), \(b\), and \(c\) satisfy the equation \(a^2 + b^2 = c^2\), then the numbers \(a\), \(b\), and \(c\) form a Pythagorean triple.

Example

a. Find \(a\).

\[
\begin{align*}
\triangle ABC & \text{ is a right triangle.} \\
\text{so } a^2 + b^2 = c^2 \\
& \text{Pythagorean Theorem} \\
a^2 + 12^2 = 13^2 & \quad b = 12, \ c = 13 \\
a^2 + 144 = 169 & \quad \text{Simplify.} \\
a^2 = 25 & \quad \text{Subtract.} \\
a = 5 & \quad \text{Take the positive square root of each side.}
\end{align*}
\]

b. Find \(c\).

\[
\begin{align*}
\triangle ABC & \text{ is a right triangle.} \\
\text{so } a^2 + b^2 = c^2 \\
& \text{Pythagorean Theorem} \\
20^2 + 30^2 = c^2 & \quad a = 20, \ b = 30 \\
400 + 900 = c^2 & \quad \text{Simplify.} \\
1300 = c^2 & \quad \text{Add.} \\
\sqrt{1300} = c & \quad \text{Take the positive square root of each side.} \\
36.1 \approx c & \quad \text{Use a calculator.}
\end{align*}
\]

Exercises

Find \(x\).

1.

2.

3.

4.

5.

6.

Use a Pythagorean Triple to find \(x\).

7.

8.

9.
Study Guide and Intervention (continued)

The Pythagorean Theorem and Its Converse

Converse of the Pythagorean Theorem If the sum of the squares of the lengths of the two shorter sides of a triangle equals the square of the lengths of the longest side, then the triangle is a right triangle.

You can also use the lengths of sides to classify a triangle.

- If \( a^2 + b^2 = c^2 \) then \( \triangle ABC \) is a right triangle.
- If \( a^2 + b^2 > c^2 \) then \( \triangle ABC \) is acute.
- If \( a^2 + b^2 < c^2 \) then \( \triangle ABC \) is obtuse.

Example Determine whether \( \triangle PQR \) is a right triangle.

\[
\begin{align*}
a^2 + b^2 &\geq c^2 \\
10^2 + (10\sqrt{3})^2 &\geq 20^2 \\
100 + 300 &\geq 400 \\
400 &\geq 400 \sqrt{3}
\end{align*}
\]

Since \( c^2 = 400 \) and \( a^2 + b^2 = 400 \), the triangle is a right triangle.

Exercises

Determine whether each set of measures can be the measures of the sides of a triangle. If so, classify the triangle as acute, obtuse, or right. Justify your answer.

1. 30, 40, 50
2. 20, 30, 40
3. 18, 24, 30
4. 6, 8, 9
5. 6, 12, 18
6. 10, 15, 20
7. \( \sqrt{5}, \sqrt{12}, \sqrt{13} \)
8. 2, \( \sqrt{8}, \sqrt{12} \)
9. 9, 40, 41
The Pythagorean Theorem and Its Converse

Find \( x \).

1. 
   \[
   \begin{array}{c}
   \text{12} \\
   \text{9} \\
   \text{x}
   \end{array}
   \]

2. 
   \[
   \begin{array}{c}
   \text{12} \\
   \text{13} \\
   \text{x}
   \end{array}
   \]

3. 
   \[
   \begin{array}{c}
   \text{12} \\
   \text{32} \\
   \text{x}
   \end{array}
   \]

4. 
   \[
   \begin{array}{c}
   \text{12.5} \\
   \text{25} \\
   \text{x}
   \end{array}
   \]

5. 
   \[
   \begin{array}{c}
   \text{8} \\
   \text{9} \\
   \text{x}
   \end{array}
   \]

6. 
   \[
   \begin{array}{c}
   \text{31} \\
   \text{14} \\
   \text{x}
   \end{array}
   \]

Use a Pythagorean Triple to find \( x \).

7. 
   \[
   \begin{array}{c}
   \text{5} \\
   \text{x} \\
   \text{12}
   \end{array}
   \]

8. 
   \[
   \begin{array}{c}
   \text{8} \\
   \text{x} \\
   \text{10}
   \end{array}
   \]

9. 
   \[
   \begin{array}{c}
   \text{12} \\
   \text{14} \\
   \text{x}
   \end{array}
   \]

10. 
   \[
   \begin{array}{c}
   \text{25} \\
   \text{x} \\
   \text{65}
   \end{array}
   \]

11. 
   \[
   \begin{array}{c}
   \text{40} \\
   \text{24} \\
   \text{x}
   \end{array}
   \]

12. 
   \[
   \begin{array}{c}
   \text{x} \\
   \text{48} \\
   \text{50}
   \end{array}
   \]

Determine whether each set of numbers can be measure of the sides of a triangle. If so, classify the triangle as \textit{acute}, \textit{obtuse}, or \textit{right}. Justify your answer.

13. 7, 24, 25

14. 8, 14, 20

15. 12.5, 13, 26

16. \(3\sqrt{2}, \sqrt{7}, 4\)

17. 20, 21, 29

18. 32, 35, 70
Find $x$.

1. \[
\begin{array}{c}
\text{13} \\
\text{23}
\end{array}
\]

2. \[
\begin{array}{c}
\text{34} \\
x \\
\text{21}
\end{array}
\]

3. \[
\begin{array}{c}
\text{26} \\
x \\
\text{18}
\end{array}
\]

4. \[
\begin{array}{c}
\text{34} \\
x \\
\text{22}
\end{array}
\]

5. \[
\begin{array}{c}
\text{16} \\
x \\
\text{14}
\end{array}
\]

6. \[
\begin{array}{c}
24 \\
x \\
24
\end{array}
\]

Use a Pythagorean Triple to find $x$.

7. \[
\begin{array}{c}
\text{36} \\
x \\
\text{27}
\end{array}
\]

8. \[
\begin{array}{c}
\text{136} \\
x \\
\text{120}
\end{array}
\]

Determine whether each set of numbers can be measure of the sides of a triangle. If so, classify the triangle as acute, obtuse, or right. Justify your answer.

11. 10, 11, 20

12. 12, 14, 49

13. $5\sqrt{2}$, 10, 11

14. 21.5, 24, 55.5

15. 30, 40, 50

16. 65, 72, 97

17. CONSTRUCTION The bottom end of a ramp at a warehouse is 10 feet from the base of the main dock and is 11 feet long. How high is the dock?
8-2 Word Problem Practice

The Pythagorean Theorem and Its Converse

1. SIDEWALKS Construction workers are building a marble sidewalk around a park that is shaped like a right triangle. Each marble slab adds 2 feet to the length of the sidewalk. The workers find that exactly 1071 and 1840 slabs are required to make the sidewalks along the short sides of the park. How many slabs are required to make the sidewalk that runs along the long side of the park?

2. RIGHT ANGLES Clyde makes a triangle using three sticks of lengths 20 inches, 21 inches, and 28 inches. Is the triangle a right triangle? Explain.

3. TETHERS To help support a flag pole, a 50-foot-long tether is tied to the pole at a point 40 feet above the ground. The tether is pulled taut and tied to an anchor in the ground. How far away from the base of the pole is the anchor?

4. FLIGHT An airplane lands at an airport 60 miles east and 25 miles north of where it took off.

   How far apart are the two airports?

5. PYTHAGOREAN TRIPLES Ms. Jones assigned her fifth-period geometry class the following problem.

   Let \( m \) and \( n \) be two positive integers with \( m > n \). Let \( a = m^2 - n^2 \), \( b = 2mn \), and \( c = m^2 + n^2 \).

   a. Show that there is a right triangle with side lengths \( a \), \( b \), and \( c \).

   b. Complete the following table.

       \[
       \begin{array}{|c|c|c|c|c|}
       \hline
       m & n & a & b & c \\
       \hline
       2 & 1 & 3 & 4 & 5 \\
       \hline
       3 & 1 & & & \\
       \hline
       3 & 2 & & & \\
       \hline
       4 & 1 & & & \\
       \hline
       4 & 2 & & & \\
       \hline
       4 & 3 & & & \\
       \hline
       5 & 1 & & & \\
       \hline
       \end{array}
       \]

   c. Find a Pythagorean triple that corresponds to a right triangle with a hypotenuse \( 25^2 = 625 \) units long. (Hint: Use the table you completed for Exercise b to find two positive integers \( m \) and \( n \) with \( m > n \) and \( m^2 + n^2 = 625 \).)
Converse of a Right Triangle Theorem

You have learned that the measure of the altitude from the vertex of the right angle of a right triangle to its hypotenuse is the geometric mean between the measures of the two segments of the hypotenuse. Is the converse of this theorem true? In order to find out, it will help to rewrite the original theorem in if-then form as follows.

If \( \triangle ABQ \) is a right triangle with right angle at \( Q \), then \( QP \) is the geometric mean between \( AP \) and \( PB \), where \( P \) is between \( A \) and \( B \) and \( QP \) is perpendicular to \( AB \).

1. Write the converse of the if-then form of the theorem.

2. Is the converse of the original theorem true? Refer to the figure at the right to explain your answer.

You may find it interesting to examine the other theorems in Chapter 8 to see whether their converses are true or false. You will need to restate the theorems carefully in order to write their converses.
Lesson 8-2

Spreadsheet Activity

Pythagorean Triples

You can use a spreadsheet to determine whether three whole numbers form a Pythagorean triple.

Example 1

Use a spreadsheet to determine whether the numbers 12, 16, and 20 form a Pythagorean triple.

Step 1

In cell A1, enter 12. In cell B1, enter 16 and in cell C1, enter 20. The longest side should be entered in column C.

Step 2

In cell D1, enter an equals sign followed by IF(A1^2+B1^2=C1^2,"YES","NO"). This will return “YES” if the set of numbers is a Pythagorean triple and will return “NO” if it is not.

The numbers 12, 16, and 20 form a Pythagorean triple.

Example 2

Use a spreadsheet to determine whether the numbers 3, 6, and 12 form a Pythagorean triple.

Step 1

In cell A2, enter 3, in cell B2, enter 6, and in cell C2, enter 12.

Step 2

Click on the bottom right corner of cell D1 and drag it to D2. This will determine whether or not the set of numbers is a Pythagorean triple.

The numbers 3, 6, and 12 do not form a Pythagorean triple.

Exercises

Use a spreadsheet to determine whether each set of numbers forms a Pythagorean triple.

1. 14, 48, 50  
2. 16, 30, 34  
3. 5, 5, 9

4. 4, 5, 7  
5. 18, 24, 30  
6. 10, 24, 26

7. 25, 60, 65  
8. 2, 4, 5  
9. 19, 21, 22

10. 18, 80, 82  
11. 5, 12, 13  
12. 20, 48, 52
Properties of 45°-45°-90° Triangles The sides of a 45°-45°-90° right triangle have a special relationship.

Example 1 If the leg of a 45°-45°-90° right triangle is $x$ units, show that the hypotenuse is $x\sqrt{2}$ units.

Using the Pythagorean Theorem with $a = b = x$, then

- $c^2 = a^2 + b^2$
- $c^2 = x^2 + x^2$
- $c^2 = 2x^2$
- $c = \sqrt{2x^2}$
- $c = x\sqrt{2}$

Example 2 In a 45°-45°-90° right triangle the hypotenuse is $\sqrt{2}$ times the leg. If the hypotenuse is 6 units, find the length of each leg.

The hypotenuse is $\sqrt{2}$ times the leg, so divide the length of the hypotenuse by $\sqrt{2}$.

$$a = \frac{6}{\sqrt{2}} = \frac{6}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{6\sqrt{2}}{2} = 3\sqrt{2} \text{ units}$$

Exercises
Find $x$.

1. \[
\begin{array}{c}
45° \quad x \\
7 \quad 45° \\
\end{array}
\]

2. \[
\begin{array}{c}
45° \quad 3\sqrt{2} \\
x \\
\end{array}
\]

3. \[
\begin{array}{c}
45° \quad 4 \\
x \\
\end{array}
\]

4. \[
\begin{array}{c}
18 \\
x \quad x \\
\end{array}
\]

5. \[
\begin{array}{c}
16 \\
x \\
\end{array}
\]

6. \[
\begin{array}{c}
24\sqrt{2} \\
x \quad 45° \\
\end{array}
\]

7. If a 45°-45°-90° triangle has a hypotenuse length of 12, find the leg length.

8. Determine the length of the leg of 45°-45°-90° triangle with a hypotenuse length of 25 inches.

9. Find the length of the hypotenuse of a 45°-45°-90° triangle with a leg length of 14 centimeters.
Properties of 30°-60°-90° Triangles The sides of a 30°-60°-90° right triangle also have a special relationship.

Example 1 In a 30°-60°-90° right triangle the hypotenuse is twice the shorter leg. Show that the longer leg is $\sqrt{3}$ times the shorter leg.

\[ \triangle MNQ \] is a 30°-60°-90° right triangle, and the length of the hypotenuse $MN$ is two times the length of the shorter side $NQ$.

Use the Pythagorean Theorem.

\[ a^2 = (2x)^2 - x^2 \]
\[ a^2 = 4x^2 - x^2 \]
\[ a^2 = 3x^2 \]
\[ a = \sqrt{3x^2} \]
\[ a = x\sqrt{3} \]

Example 2 In a 30°-60°-90° right triangle, the hypotenuse is 5 centimeters. Find the lengths of the other two sides of the triangle.

If the hypotenuse of a 30°-60°-90° right triangle is 5 centimeters, then the length of the shorter leg is one-half of 5, or 2.5 centimeters. The length of the longer leg is $\sqrt{3}$ times the length of the shorter leg, or $(2.5)(\sqrt{3})$ centimeters.

Exercises

Find $x$ and $y$.

1.

2.

3.

4.

5.

6.

7. An equilateral triangle has an altitude length of 36 feet. Determine the length of a side of the triangle.

8. Find the length of the side of an equilateral triangle that has an altitude length of 45 centimeters.
8-3 Skills Practice

Special Right Triangles

Find $x$.

1. \[
\begin{array}{c}
\text{45°} \\
\text{25} \\
x
\end{array}
\]

2. \[
\begin{array}{c}
\text{45°} \\
x \\
17
\end{array}
\]

3. \[
\begin{array}{c}
\text{45°} \\
48 \\
x
\end{array}
\]

4. \[
\begin{array}{c}
\text{45°} \\
x \\
100
\end{array}
\]

5. \[
\begin{array}{c}
\text{45°} \\
x \\
100
\end{array}
\]

6. \[
\begin{array}{c}
\text{45°} \\
88 \\
x
\end{array}
\]

7. Determine the length of the leg of 45°-45°-90° triangle with a hypotenuse length of 26.

8. Find the length of the hypotenuse of a 45°-45°-90° triangle with a leg length of 50 centimeters.

Find $x$ and $y$.

9. \[
\begin{array}{c}
\text{30°} \\
11 \\
x \\
y
\end{array}
\]

10. \[
\begin{array}{c}
\text{60°} \\
x \\
8\sqrt{3}
\end{array}
\]

11. \[
\begin{array}{c}
\text{30°} \\
x \\
y \\
5\sqrt{3}
\end{array}
\]

12. \[
\begin{array}{c}
\text{30°} \\
x \\
y \\
30
\end{array}
\]

13. \[
\begin{array}{c}
\text{30°} \\
x \\
y \\
21\sqrt{3}
\end{array}
\]

14. \[
\begin{array}{c}
\text{30°} \\
x \\
y \\
52\sqrt{3}
\end{array}
\]

15. An equilateral triangle has an altitude length of 27 feet. Determine the length of a side of the triangle.

16. Find the length of the side of an equilateral triangle that has an altitude length of $11\sqrt{3}$ feet.
Find $x$. 

1. \[
\begin{array}{c}
\text{45°} \\
14
\end{array}
\]
2. \[
\begin{array}{c}
\text{45°} \\
45
\end{array}
\]
3. \[
\begin{array}{c}
\text{45°} \\
22
\end{array}
\]
4. \[
\begin{array}{c}
\text{45°} \\
210
\end{array}
\]
5. \[
\begin{array}{c}
\text{45°} \\
88
\end{array}
\]
6. \[
\begin{array}{c}
\text{45°} \\
5\sqrt{2}
\end{array}
\]

Find $x$ and $y$. 

7. \[
\begin{array}{c}
\text{30°} \\
9 \\
30°  \\
y
\end{array}
\]
8. \[
\begin{array}{c}
\text{60°} \\
4\sqrt{3} \\
x  \\
y
\end{array}
\]
9. \[
\begin{array}{c}
\text{30°} \\
x \\
y  \\
20
\end{array}
\]
10. \[
\begin{array}{c}
\text{60°} \\
x  \\
y  \\
98
\end{array}
\]

11. Determine the length of the leg of 45°-45°-90° triangle with a hypotenuse length of 38.

12. Find the length of the hypotenuse of a 45°-45°-90° triangle with a leg length of 77 centimeters.

13. An equilateral triangle has an altitude length of 33 feet. Determine the length of a side of the triangle.

14. **BOTANICAL GARDENS** One of the displays at a botanical garden is an herb garden planted in the shape of a square. The square measures 6 yards on each side. Visitors can view the herbs from a diagonal pathway through the garden. How long is the pathway?
1. **ORIGAMI** A square piece of paper 150 millimeters on a side is folded in half along a diagonal. The result is a 45°-45°-90° triangle. What is the length of the hypotenuse of this triangle?

2. **ESCALATORS** A 40-foot-long escalator rises from the first floor to the second floor of a shopping mall. The escalator makes a 30° angle with the horizontal. How high above the first floor is the second floor?

3. **HEXAGONS** A box of chocolates shaped like a regular hexagon is placed snugly inside of a rectangular box as shown in the figure. If the side length of the hexagon is 3 inches, what are the dimensions of the rectangular box?

4. **WINDOWS** A large stained glass window is constructed from six 30°-60°-90° triangles as shown in the figure. What is the height of the window?

5. **MOVIES** Kim and Yolanda are watching a movie in a movie theater. Yolanda is sitting $x$ feet from the screen and Kim is 15 feet behind Yolanda.

   a. How high is the top of the screen in terms of $x$?

   b. What is $\frac{x + 15}{x}$?

   c. How far is Yolanda from the screen? Round your answer to the nearest tenth.
Constructing Values of Square Roots

The diagram at the right shows a right isosceles triangle with two legs of length 1 inch. By the Pythagorean Theorem, the length of the hypotenuse is \( \sqrt{2} \) inches. By constructing an adjacent right triangle with legs of \( \sqrt{2} \) inches and 1 inch, you can create a segment of length \( \sqrt{3} \).

By continuing this process as shown below, you can construct a “wheel” of square roots. This wheel is called the “Wheel of Theodorus” after a Greek philosopher who lived about 400 B.C.

Continue constructing the wheel until you make a segment of length \( \sqrt{18} \).
Trigonometric Ratios The ratio of the lengths of two sides of a right triangle is called a trigonometric ratio. The three most common ratios are sine, cosine, and tangent, which are abbreviated $\sin$, $\cos$, and $\tan$, respectively.

\[
\sin R = \frac{\text{leg opposite } \angle R}{\text{hypotenuse}} \quad \cos R = \frac{\text{leg adjacent to } \angle R}{\text{hypotenuse}} \quad \tan R = \frac{\text{leg opposite } \angle R}{\text{leg adjacent to } \angle R}
\]

\[
= \frac{r}{t} \quad \quad = \frac{s}{t} \quad \quad = \frac{r}{s}
\]

**Example** Find $\sin A$, $\cos A$, and $\tan A$. Express each ratio as a fraction and a decimal to the nearest hundredth.

\[
\sin A = \frac{\text{opposite leg}}{\text{hypotenuse}} = \frac{BC}{BA} = \frac{5}{13} \approx 0.38
\]

\[
\cos A = \frac{\text{adjacent leg}}{\text{hypotenuse}} = \frac{AC}{AB} = \frac{12}{13} \approx 0.92
\]

\[
\tan A = \frac{\text{opposite leg}}{\text{adjacent leg}} = \frac{BC}{AC} = \frac{5}{12} \approx 0.42
\]

**Exercises**

Find $\sin J$, $\cos J$, $\tan J$, $\sin L$, $\cos L$, and $\tan L$. Express each ratio as a fraction and as a decimal to the nearest hundredth if necessary.

1. \[\begin{align*}
J & \quad 16 \\
K & \quad 12 \\
L & \quad 20
\end{align*}\]

2. \[\begin{align*}
J & \quad 32 \\
K & \quad 24 \\
L & \quad 40
\end{align*}\]

3. \[\begin{align*}
J & \quad 12\sqrt{3} \\
K & \quad 36 \\
L & \quad 24\sqrt{3}
\end{align*}\]
Use Inverse Trigonometric Ratios  You can use a calculator and the sine, cosine, or tangent to find the measure of the angle, called the inverse of the trigonometric ratio.

Example  Use a calculator to find the measure of $\angle T$ to the nearest tenth.

The measures given are those of the leg opposite $\angle T$ and the hypotenuse, so write an equation using the sine ratio.

$$\sin T = \frac{\text{opp}}{\text{hyp}} \quad \sin T = \frac{29}{34}$$

If $\sin T = \frac{29}{34}$, then $\sin^{-1} \frac{29}{34} = m \angle T$.

Use a calculator. So, $m \angle T \approx 58.5$. 

Exercises

Use a calculator to find the measure of $\angle T$ to the nearest tenth.

1. \[ \begin{align*}
S \quad & 14\sqrt{3} \\
R \quad & 34 \\
T & \\
\end{align*} \]

2. \[ \begin{align*}
S \quad & 7 \\
R \quad & 18 \\
T & \\
\end{align*} \]

3. \[ \begin{align*}
S \quad & 34 \\
R \quad & 57 \\
T & \\
\end{align*} \]

4. \[ \begin{align*}
S \quad & 32 \\
R \quad & 101 \\
T & \\
\end{align*} \]

5. \[ \begin{align*}
S \quad & 10\sqrt{3} \\
R \quad & 67 \\
T & \\
\end{align*} \]

6. \[ \begin{align*}
S \quad & 14\sqrt{2} \\
R \quad & 39 \\
T & \\
\end{align*} \]
Skills Practice

Trigonometry

Find \( R \), \( \cos R \), \( \tan R \), \( S \), \( \cos S \), and \( \tan S \).
Express each ratio as a fraction and as a decimal to the nearest hundredth.

1. \( r = 16, s = 30, t = 34 \)
2. \( r = 10, s = 24, t = 26 \)

Use a special right triangle to express each trigonometric ratio as a fraction and as a decimal to the nearest hundredth if necessary.

3. \( \sin 30^\circ \)
4. \( \tan 45^\circ \)
5. \( \cos 60^\circ \)
6. \( \sin 60^\circ \)
7. \( \tan 30^\circ \)
8. \( \cos 45^\circ \)

Find \( x \). Round to the nearest hundredth if necessary.

9.

10.

11.

Use a calculator to find the measure of \( \angle B \) to the nearest tenth.

12.

13.

14.
Find $\sin L$, $\cos L$, $\tan L$, $\sin M$, $\cos M$, and $\tan M$.
Express each ratio as a fraction and as a decimal to the nearest hundredth.

1. $\ell = 15, m = 36, n = 39$
2. $\ell = 12, m = 12\sqrt{3}, n = 24$

Find $x$. Round to the nearest hundredth.

3. 
4. 
5. 

Use a calculator to find the measure of $\angle B$ to the nearest tenth.

6. 
7. 
8. 

9. GEOGRAPHY Diego used a theodolite to map a region of land for his class in geomorphology. To determine the elevation of a vertical rock formation, he measured the distance from the base of the formation to his position and the angle between the ground and the line of sight to the top of the formation. The distance was 43 meters and the angle was 36°. What is the height of the formation to the nearest meter?
1. **RADIO TOWERS** Kay is standing near a 200-foot-high radio tower.

Use the information in the figure to determine how far Kay is from the top of the tower. Express your answer as a trigonometric function.

2. **RAMPS** A 60-foot ramp rises from the first floor to the second floor of a parking garage. The ramp makes a 15° angle with the ground.

How high above the first floor is the second floor? Express your answer as a trigonometric function.

3. **TRIGONOMETRY** Melinda and Walter were both solving the same trigonometry problem. However, after they finished their computations, Melinda said the answer was 52 sin 27° and Walter said the answer was 52 cos 63°. Could they both be correct? Explain.

4. **LINES** Jasmine draws line \( m \) on a coordinate plane.

What angle does \( m \) make with the \( x \)-axis? Round your answer to the nearest degree.

5. **NEIGHBORS** Amy, Barry, and Chris live on the same block. Chris lives up the street and around the corner from Amy, and Barry lives at the corner between Amy and Chris. The three homes are the vertices of a right triangle.

a. Give two trigonometric expressions for the ratio of Barry’s distance from Amy to Chris’ distance from Amy.

b. Give two trigonometric expressions for the ratio of Barry’s distance from Chris to Amy’s distance from Chris.

c. Give a trigonometric expression for the ratio of Amy’s distance from Barry to Chris’ distance from Barry.
Sine and Cosine of Angles

The following diagram can be used to obtain approximate values for the sine and cosine of angles from 0° to 90°. The radius of the circle is 1. So, the sine and cosine values can be read directly from the vertical and horizontal axes.

Example

Find approximate values for \( \sin 40° \) and \( \cos 40° \). Consider the triangle formed by the segment marked 40°, as illustrated by the shaded triangle at right.

\[
\sin 40° = \frac{a}{c} \approx \frac{0.64}{1} \text{ or } 0.64 \quad \cos 40° = \frac{b}{c} \approx \frac{0.77}{1} \text{ or } 0.77
\]

1. Use the diagram above to complete the chart of values.

<table>
<thead>
<tr>
<th>( x^\circ )</th>
<th>0°</th>
<th>10°</th>
<th>20°</th>
<th>30°</th>
<th>40°</th>
<th>50°</th>
<th>60°</th>
<th>70°</th>
<th>80°</th>
<th>90°</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sin x^\circ )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.64</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \cos x^\circ )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.77</td>
<td></td>
</tr>
</tbody>
</table>

2. Compare the sine and cosine of two complementary angles (angles with a sum of 90°). What do you notice?
**Angles of Elevation and Depression**

Many real-world problems that involve looking up to an object can be described in terms of an **angle of elevation**, which is the angle between an observer’s line of sight and a horizontal line.

When an observer is looking down, the **angle of depression** is the angle between the observer’s line of sight and a horizontal line.

**Example**

The angle of elevation from point A to the top of a cliff is 34°. If point A is 1000 feet from the base of the cliff, how high is the cliff?

Let \( x \) = the height of the cliff.

\[
\tan 34° = \frac{x}{1000} \quad \text{tan} = \frac{\text{opposite}}{\text{adjacent}}
\]

\[
1000(\tan 34°) = x \quad \text{Multiply each side by 1000.}
\]

\[
674.5 \approx x \quad \text{Use a calculator.}
\]

The height of the cliff is about 674.5 feet.

**Exercises**

1. **HILL TOP** The angle of elevation from point A to the top of a hill is 49°. If point A is 400 feet from the base of the hill, how high is the hill?

2. **SUN** Find the angle of elevation of the Sun when a 12.5-meter-tall telephone pole casts an 18-meter-long shadow.

3. **SKIING** A ski run is 1000 yards long with a vertical drop of 208 yards. Find the angle of depression from the top of the ski run to the bottom.

4. **AIR TRAFFIC** From the top of a 120-foot-high tower, an air traffic controller observes an airplane on the runway at an angle of depression of 19°. How far from the base of the tower is the airplane?
Two Angles of Elevation or Depression  Angles of elevation or depression to two different objects can be used to estimate distance between those objects. The angles from two different positions of observation to the same object can be used to estimate the height of the object.

**Example**  To estimate the height of a garage, Jason sights the top of the garage at a 42° angle of elevation. He then steps back 20 feet and sights the top at a 10° angle. If Jason is 6 feet tall, how tall is the garage to the nearest foot?

\( \triangle ABC \) and \( \triangle ABD \) are right triangles. We can determine \( AB = x \) and \( CB = y \), and \( DB = y + 20 \).

Use \( \triangle ABC \).  Use \( \triangle ABD \).

\[
\tan 42^\circ = \frac{x}{y} \quad \text{or} \quad y \tan 42^\circ = x
\]

\[
\tan 10^\circ = \frac{x}{y + 20} \quad \text{or} \quad (y + 20) \tan 10^\circ = x
\]

Substitute the value for \( x \) from \( \triangle ABD \) in the equation for \( \triangle ABC \) and solve for \( y \).

\[
y \tan 42^\circ = (y + 20) \tan 10^\circ
\]

\[
y \tan 42^\circ = y \tan 10^\circ + 20 \tan 10^\circ
\]

\[
y \tan 42^\circ - y \tan 10^\circ = 20 \tan 10^\circ
\]

\[
y (\tan 42^\circ - \tan 10^\circ) = 20 \tan 10^\circ
\]

\[
y = \frac{20 \tan 10^\circ}{\tan 42^\circ - \tan 10^\circ} \approx 4.87
\]

If \( y \approx 4.87 \), then \( x = 4.87 \tan 42^\circ \) or about 4.4 feet. Add Jason’s height, so the garage is about 4.4 + 6 or 10.4 feet tall.

**Exercises**

1. **CLIFF**  Sarah stands on the ground and sights the top of a steep cliff at a 60° angle of elevation. She then steps back 50 meters and sights the top of the steep cliff at a 30° angle. If Sarah is 1.8 meters tall, how tall is the steep cliff to the nearest meter?

2. **BALLOON**  The angle of depression from a hot air balloon in the air to a person on the ground is 36°. If the person steps back 10 feet, the new angle of depression is 25°. If the person is 6 feet tall, how far off the ground is the hot air balloon?
5. MOUNTAIN BIKING  On a mountain bike trip along the Gemini Bridges Trail in Moab, Utah, Nabuko stopped on the canyon floor to get a good view of the twin sandstone bridges. Nabuko is standing about 60 meters from the base of the canyon cliff, and the natural arch bridges are about 100 meters up the canyon wall. If her line of sight is 5 meters above the ground, what is the angle of elevation to the top of the bridges? Round to the nearest tenth degree.

6. SHADOWS  Suppose the sun casts a shadow off a 35-foot building. If the angle of elevation to the sun is $60^\circ$, how long is the shadow to the nearest tenth of a foot?

7. BALLOONING  Angie sees a hot air balloon in the sky from her spot on the ground. The angle of elevation from Angie to the balloon is $40^\circ$. If she steps back 200 feet, the new angle of elevation is $10^\circ$. If Angie is 5.5 feet tall, how far off the ground is the hot air balloon?

8. INDIRECT MEASUREMENT  Kyle is at the end of a pier 30 feet above the ocean. His eye level is 3 feet above the pier. He is using binoculars to watch a whale surface. If the angle of depression of the whale is $20^\circ$, how far is the whale from Kyle’s binoculars? Round to the nearest tenth foot.
Name the angle of depression or angle of elevation in each figure.

1. 

2. 

3. **WATER TOWERS** A student can see a water tower from the closest point of the soccer field at San Lobos High School. The edge of the soccer field is about 110 feet from the water tower and the water tower stands at a height of 32.5 feet. What is the angle of elevation if the eye level of the student viewing the tower from the edge of the soccer field is 6 feet above the ground? Round to the nearest tenth.

4. **CONSTRUCTION** A roofer props a ladder against a wall so that the top of the ladder reaches a 30-foot roof that needs repair. If the angle of elevation from the bottom of the ladder to the roof is 55°, how far is the ladder from the base of the wall? Round your answer to the nearest foot.

5. **TOWN ORDINANCES** The town of Belmont restricts the height of flagpoles to 25 feet on any property. Lindsay wants to determine whether her school is in compliance with the regulation. Her eye level is 5.5 feet from the ground and she stands 36 feet from the flagpole. If the angle of elevation is about 25°, what is the height of the flagpole to the nearest tenth?

6. **GEOGRAPHY** Stephan is standing on the ground by a mesa in the Painted Desert. Stephan is 1.8 meters tall and sights the top of the mesa at 29°. Stephan steps back 100 meters and sights the top at 25°. How tall is the mesa?

7. **INDIRECT MEASUREMENT** Mr. Dominguez is standing on a 40-foot ocean bluff near his home. He can see his two dogs on the beach below. If his line of sight is 6 feet above the ground and the angles of depression to his dogs are 34° and 48°, how far apart are the dogs to the nearest foot?
1. **LIGHTHOUSES** Sailors on a ship at sea spot the light from a lighthouse. The angle of elevation to the light is 25°.

The light of the lighthouse is 30 meters above sea level. How far from the shore is the ship? Round your answer to the nearest meter.

2. **RESCUE** A hiker dropped his backpack over one side of a canyon onto a ledge below. Because of the shape of the cliff, he could not see exactly where it landed.

From the other side, the park ranger reports that the angle of depression to the backpack is 32°. If the width of the canyon is 115 feet, how far down did the backpack fall? Round your answer to the nearest foot.

3. **AIRPLANES** The angle of elevation to an airplane viewed from the control tower at an airport is 7°. The tower is 200 feet high and the pilot reports that the altitude is 5200 feet. How far away from the control tower is the airplane? Round your answer to the nearest foot.

4. **PEAK TRAM** The Peak Tram in Hong Kong connects two terminals, one at the base of a mountain, and the other at the summit. The angle of elevation of the upper terminal from the lower terminal is about 15.5°. The distance between the two terminals is about 1365 meters. About how much higher above sea level is the upper terminal compared to the lower terminal? Round your answer to the nearest meter.

5. **HELICOPTERS** Jermaine and John are watching a helicopter hover above the ground.

Jermaine and John are standing 10 meters apart.

**a.** Find two different expressions that can be used to find the height of the helicopter.

**b.** Equate the two expressions you found for Exercise a to solve for \(x\). Round your answer to the nearest hundredth.

**c.** How high above the ground is the helicopter? Round your answer to the nearest hundredth.
**Best Seat in the House**

Most people want to sit in the best seat in the movie theater. The best seat could be defined as the seat that allows you to see the maximum amount of screen. The picture below represents this situation.

To determine the best seat in the house, you want to find what value of $x$ allows you to see the maximum amount of screen. The value of $x$ is how far from the screen you should sit.

1. To maximize the amount of screen viewed, which angle value needs to be maximized? Why?

2. What is the value of $a$ if $x = 10$ feet?

3. What is the value of $a$ if $x = 20$ feet?

4. What is the value of $a$ if $x = 25$ feet?

5. What is the value of $a$ if $x = 35$ feet?

6. What is the value of $a$ if $x = 55$ feet?

7. Which value of $x$ gives the greatest value of $a$? So, where is the best seat in the movie theater?
The Law of Sines

In any triangle, there is a special relationship between the angles of the triangle and the lengths of the sides opposite the angles.

The Law of Sines

\[
\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}
\]

**Example 1**

Find \(b\). Round to the nearest tenth.

\[
\frac{\sin 45°}{30} = \frac{\sin 74°}{b} 
\]

\[
b = \frac{30 \sin 74°}{\sin 45°}
\]

\[
b \approx 40.8
\]

**Example 2**

Find \(d\). Round to the nearest tenth.

By the Triangle Angle-Sum Theorem, \(m\angle E = 180° - (82° + 40°)\) or 58.

\[
\frac{\sin D}{d} = \frac{\sin 82°}{\sin 58°}
\]

\[
d = \frac{24 \sin 82°}{\sin 58°}
\]

\[
d \approx 28.0
\]

**Exercises**

Find \(x\). Round to the nearest tenth.

1. \[
\begin{align*}
80° & \quad 40° \\
12 & \quad x
\end{align*}
\]

2. \[
\begin{align*}
20° & \quad 91° \\
x & \quad 52°
\end{align*}
\]

3. \[
\begin{align*}
34° & \quad 16 \\
x & \quad 84°
\end{align*}
\]

4. \[
\begin{align*}
17° & \quad 72° \\
x & \quad 25
\end{align*}
\]

5. \[
\begin{align*}
12 & \quad 89° \\
x & \quad 80°
\end{align*}
\]

6. \[
\begin{align*}
35° & \quad 35° \\
x & \quad 60°
\end{align*}
\]
The Law of Cosines

Another relationship between the sides and angles of any triangle is called the **Law of Cosines**. You can use the Law of Cosines if you know three sides of a triangle or if you know two sides and the included angle of a triangle.

### Law of Cosines

Let \( \triangle ABC \) be any triangle with \( a, b, \) and \( c \) representing the measures of the sides opposite the angles with measures \( A, B, \) and \( C \), respectively. Then the following equations are true.

\[
\begin{align*}
    a^2 &= b^2 + c^2 - 2bc \cos A \\
    b^2 &= a^2 + c^2 - 2ac \cos B \\
    c^2 &= a^2 + b^2 - 2ab \cos C 
\end{align*}
\]

**Example 1** Find \( c \). Round to the nearest tenth.

\[
c = \sqrt{a^2 + b^2 - 2ab \cos C}
\]

\[
c = \sqrt{12^2 + 10^2 - 2(12)(10)\cos 48^\circ}
\]

\[
c \approx 9.1
\]

**Example 2** Find \( m\angle A \). Round to the nearest degree.

\[
a^2 = b^2 + c^2 - 2bc \cos A
\]

\[
a = 7, b = 5, c = 8
\]

\[
7^2 = 5^2 + 8^2 - 2(5)(8) \cos A
\]

\[
49 = 25 + 64 - 80 \cos A
\]

\[
-40 = -80 \cos A
\]

\[
\frac{1}{2} = \cos A
\]

\[
\cos^{-1} \frac{1}{2} = A
\]

\[
60^\circ = A
\]

---

**Exercises**

Find \( x \). Round angle measures to the nearest degree and side measures to the nearest tenth.

1. ![Diagram](image1)

2. ![Diagram](image2)

3. ![Diagram](image3)

4. ![Diagram](image4)

5. ![Diagram](image5)

6. ![Diagram](image6)
8-6 Skills Practice

The Law of Sines and Law of Cosines

Find x. Round angle measures to the nearest degree and side lengths to the nearest tenth.

1. \( \triangle ABC \)
   - \( A = 35° \)
   - \( B = 48° \)
   - \( C = 28 \)

2. \( \triangle ABC \)
   - \( A = 18 \)
   - \( B = 17° \)
   - \( C = 46° \)

3. \( \triangle ABC \)
   - \( A = 86° \)
   - \( B = 38 \)
   - \( C = 51° \)

4. \( \triangle ABC \)
   - \( A = 73° \)
   - \( B = 80° \)
   - \( C = 8 \)

5. \( \triangle ABC \)
   - \( A = 41° \)
   - \( B = 88° \)
   - \( C = 34 \)

6. \( \triangle ABC \)
   - \( A = 60° \)
   - \( B = 12 \)
   - \( C = 83° \)

7. \( \triangle ABC \)
   - \( A = 22° \)
   - \( B = 104° \)
   - \( C = 18 \)

8. \( \triangle PQR \)
   - \( P = 54° \)
   - \( Q = 27 \)
   - \( R = 93° \)

9. \( \triangle PQR \)
   - \( P = 11° \)
   - \( Q = 12 \)
   - \( R = 54° \)

10. \( \triangle PQR \)
    - \( P = 34 \)
    - \( Q = 16 \)
    - \( R = 99° \)

11. \( \triangle PQR \)
    - \( P = 13 \)
    - \( Q = 20 \)
    - \( R = 103° \)

12. \( \triangle PQR \)
    - \( P = 13 \)
    - \( Q = 12 \)
    - \( R = 28° \)

13. \( \triangle PQR \)
    - \( P = 35 \)
    - \( Q = 13 \)
    - \( R = 96° \)

14. \( \triangle PQR \)
    - \( P = 26 \)
    - \( Q = 28 \)
    - \( R = 30 \)

15. \( \triangle PQR \)
    - \( P = 20 \)
    - \( Q = 25 \)
    - \( R = 28° \)

16. Solve the triangle. Round angle measures to the nearest degree.

   \( \triangle ABC \)
   - \( A = 18 \)
   - \( B = 23 \)
   - \( C = 17 \)
13. **INDIRECT MEASUREMENT** To find the distance from the edge of the lake to the tree on the island in the lake, Hannah set up a triangular configuration as shown in the diagram. The distance from location $A$ to location $B$ is 85 meters. The measures of the angles at $A$ and $B$ are $51^\circ$ and $83^\circ$, respectively. What is the distance from the edge of the lake at $B$ to the tree on the island at $C$?
1. ALTITUDES In triangle $ABC$, the altitude to side $AB$ is drawn.

Give two expressions for the length of the altitude in terms of $a$, $b$, and the sine of the angles $A$ and $B$.

2. MAPS Three cities form the vertices of a triangle. The angles of the triangle are $40^\circ$, $60^\circ$, and $80^\circ$. The two most distant cities are 40 miles apart. How close are the two closest cities? Round your answer to the nearest tenth of a mile.

3. STATUES Gail was visiting an art gallery. In one room, she stood so that she had a view of two statues, one of a man, and the other of a woman. She was 40 feet from the statue of the woman, and 35 feet from the statue of the man. The angle created by the lines of sight to the two statues was $21^\circ$. What is the distance between the two statues? Round your answer to the nearest tenth.

4. CARS Two cars start moving from the same location. They head straight, but in different directions. The angle between where they are heading is $43^\circ$. The first car travels 20 miles and the second car travels 37 miles. How far apart are the two cars? Round your answer to the nearest tenth.

5. ISLANDS Oahu is a Hawaiian Island. Off the coast of Oahu, there is a very tiny island known as Chinaman’s Hat. Keoki and Malia are observing Chinaman’s Hat from locations 5 kilometers apart. Use the information in the figure to answer the following questions.

a. How far is Keoki from Chinaman’s Hat? Round your answer to the nearest tenth of a kilometer.

b. How far is Malia from Chinaman’s Hat? Round your answer to the nearest tenth of a kilometer.
Identities

An identity is an equation that is true for all values of the variable for which both sides are defined. One way to verify an identity is to use a right triangle and the definitions for trigonometric functions.

Example 1  Verify that \((\sin A)^2 + (\cos A)^2 = 1\) is an identity.

\[
(\sin A)^2 + (\cos A)^2 = \left(\frac{a}{c}\right)^2 + \left(\frac{b}{c}\right)^2 = \frac{a^2 + b^2}{c^2} = \frac{c^2}{c^2} = 1
\]

To check whether an equation may be an identity, you can test several values. However, since you cannot test all values, you cannot be certain that the equation is an identity.

Example 2  Test \(\sin 2x = 2 \sin x \cos x\) to see if it could be an identity.

Try \(x = 20\). Use a calculator to evaluate each expression.

\[
\sin 2x = \sin 40 \approx 0.643 \\
2 \sin x \cos x = 2 (\sin 20)(\cos 20) \approx 2(0.342)(0.940) \approx 0.643
\]

Since the left and right sides seem equal, the equation may be an identity.

Exercises

Use triangle \(ABC\) shown above. Verify that each equation is an identity.

1. \(\frac{\cos A}{\sin A} = \frac{1}{\tan A}\)
2. \(\frac{\tan B}{\sin B} = \frac{1}{\cos B}\)

3. \(\tan B \cos B = \sin B\)
4. \(1 - (\cos B)^2 = (\sin B)^2\)

Try several values for \(x\) to test whether each equation could be an identity.

5. \(\cos 2x = (\cos x)^2 - (\sin x)^2\)
6. \(\cos (90 - x) = \sin x\)
Graphing Calculator Activity

Solving Triangles Using the Law of Sines or Cosines

You can use a calculator to solve triangles using the Law of Sines or Cosines.

Example

Solve \( \triangle ABC \) if \( a = 6 \), \( b = 2 \), and \( c = 7.5 \).

Use the Law of Cosines.

\[
\begin{align*}
    a^2 &= b^2 + c^2 - 2bc \cos A \\
    6^2 &= 2^2 + 7.5^2 - 2(2)(7.5) \cos A \\
    m\angle A &= \cos^{-1}\left( \frac{6^2 - 2^2 - 7.5^2}{-2(2)(7.5)} \right)
\end{align*}
\]

Use your calculator to find the measure of \( \angle A \).

Keystrokes:

\[
\text{2nd} \quad [\text{COS}] \quad 6 \quad \times \quad 2 \quad \times \quad 7.5 \quad ) \quad \div \quad 36.06658826
\]

So \( m\angle A \approx 36 \). Use the Law of Sines and your calculator to find \( m\angle B \).

\[
\frac{\sin A}{a} = \frac{\sin B}{b}
\]

\[
\frac{\sin 36}{6} \approx \frac{\sin B}{2}
\]

\[
m\angle B \approx \sin^{-1}\left( \frac{2 \sin 36}{6} \right)
\]

Keystrokes:

\[
\text{2nd} \quad [\text{SIN}] \quad 2 \quad \text{SIN} \quad 36 \quad ) \quad \div \quad 6 \quad ) \quad \text{ENTER} \quad 11.29896425
\]

So \( m\angle B \approx 11 \). By the Triangle Angle-Sum Theorem, \( m\angle C \approx 180 - (36 + 11) \) or 133.

Exercises

Solve each \( \triangle ABC \). Round measures of sides to the nearest tenth and measures of angles to the nearest degree.

1. \( a = 9 \), \( b = 14 \), \( c = 12 \)

2. \( m\angle C = 80 \), \( c = 9 \), \( m\angle A = 40 \)

3. \( m\angle B = 45 \), \( m\angle C = 56 \), \( a = 2 \)

4. \( a = 5.7 \), \( b = 6 \), \( c = 5 \)

5. \( a = 11 \), \( b = 15 \), \( c = 21 \)
### Study Guide and Intervention

#### Vectors

**Geometric Vector Operations** A vector is a directed segment representing a quantity that has both **magnitude**, or length, and **direction**. For example, the speed and direction of an airplane can be represented by a vector. In symbols, a vector is written as \( \overrightarrow{AB} \), where \( A \) is the initial point and \( B \) is the endpoint, or as \( \vec{v} \). The sum of two vectors is called the **resultant**. Subtracting a vector is equivalent to adding its opposite. The resultant of two vectors can be found using the **parallelogram method** or the **triangle method**.

#### Example

Copy the vectors to find \( \vec{a} - \vec{b} \).

**Method 1:** Use the parallelogram method.

<table>
<thead>
<tr>
<th>Copy ( \vec{a} ) and ( -\vec{b} ) with the same initial point.</th>
<th>Complete the parallelogram.</th>
<th>Draw the diagonal of the parallelogram from the initial point.</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \vec{a} )</td>
<td>( \vec{a} )</td>
<td>( \vec{a} - \vec{b} )</td>
</tr>
<tr>
<td>( -\vec{b} )</td>
<td>( \vec{a} - \vec{b} )</td>
<td></td>
</tr>
</tbody>
</table>

**Method 2:** Use the triangle method.

<table>
<thead>
<tr>
<th>Copy ( \vec{a} ).</th>
<th>Place the initial point of ( -\vec{b} ) at the terminal point of ( \vec{a} ).</th>
<th>Draw the vector from the initial point of ( \vec{a} ) to the terminal point of ( -\vec{b} ).</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \vec{a} )</td>
<td>( \vec{a} - \vec{b} )</td>
<td></td>
</tr>
<tr>
<td>( \vec{a} )</td>
<td>( \vec{a} - \vec{b} )</td>
<td></td>
</tr>
</tbody>
</table>

#### Exercises

Copy the vectors. Then find each sum or difference.

1. \( \vec{c} + \vec{d} \)

2. \( \vec{w} - \vec{z} \)

3. \( \vec{a} - \vec{b} \)

4. \( \vec{r} + \vec{t} \)
Study Guide and Intervention (continued)

8-7 Vectors

Vectors on the Coordinate Plane  A vector in **standard position** has its initial point at (0, 0) and can be represented by the ordered pair for point $B$. The vector at the right can be expressed as $\vec{v} = \langle 5, 3 \rangle$.

You can use the Distance Formula to find the magnitude $|\overrightarrow{AB}|$ of a vector. You can describe the direction of a vector by measuring the angle that the vector forms with the positive $x$-axis or with any other horizontal line.

**Example**  Find the magnitude and direction of $\vec{a} = \langle 3, 5 \rangle$.

Find the magnitude.

$$\vec{a} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(3 - 0)^2 + (5 - 0)^2}$$

$$= \sqrt{34} \text{ or about 5.8}$$

To find the direction, use the tangent ratio.

$$\tan \theta = \frac{5}{3}$$

The tangent ratio is opposite over adjacent.

$$m \angle \theta \approx 59.0$$

Use a calculator.

The magnitude of the vector is about 5.8 units and its direction is $59^\circ$.

**Exercises**

Find the magnitude and direction of each vector.

1. $\vec{b} = \langle -5, 2 \rangle$

2. $\vec{c} = \langle -2, 1 \rangle$

3. $\vec{d} = \langle 3, 4 \rangle$

4. $\vec{m} = \langle 5, -1 \rangle$

5. $\vec{r} = \langle -3, -4 \rangle$

6. $\vec{v} = \langle -4, 1 \rangle$
8-7 Skills Practice

Vectors

Use a ruler and a protractor to draw each vector. Include a scale on each diagram.

1. \( \vec{a} \) = 20 meters per second 60° west of south

2. \( \vec{b} \) = 10 pound of force at 135° to the horizontal

Copy the vectors to find each sum or difference.

3. \( \vec{a} + \vec{z} \)

4. \( \vec{t} - \vec{r} \)

Write the component form of each vector.

5. \( \vec{m} = \langle 2, 12 \rangle \)

6. \( \vec{p} = \langle 3, 10 \rangle \)

Find the magnitude and direction of each vector.

7. \( \vec{k} = \langle -8, -3 \rangle \)

8. \( \vec{f} = \langle -5, 11 \rangle \)

Find each of the following for \( \vec{a} = \langle 2, 4 \rangle \), \( \vec{b} = \langle 3, -3 \rangle \), and \( \vec{c} = \langle 4, -1 \rangle \). Check your answers graphically.

Overmatter
8-7 Practice

Vectors

Use a ruler and a protractor to draw each vector. Include a scale on each diagram.

1. \( \vec{v} = 12 \text{ Newtons of force at } \) 40° to the horizontal
2. \( \vec{w} = 15 \text{ miles per hour } \) 70° east of north

![Diagram showing \( \vec{v} \) and \( \vec{w} \)]

Copy the vectors to find each sum or difference.

3. \( \vec{p} + \vec{r} \)
4. \( \vec{a} - \vec{b} \)

5. Write the component form of \( \vec{AB} \).

Find the magnitude and direction of each vector.

6. \( \vec{t} = (6, 11) \)
7. \( \vec{g} = (9, -7) \)

Find each of the following for \( \vec{a} = (-1.5, 4) \), \( \vec{b} = (7, 3) \), and \( \vec{c} = (1, -2) \). Check your answers graphically.

8. \( 2\vec{a} + \vec{b} \)
9. \( 2\vec{c} - \vec{b} \)

10. AVIATION A jet begins a flight along a path due north at 300 miles per hour. A wind is blowing due west at 30 miles per hour.
   a. Find the resultant velocity of the plane.
   b. Find the resultant direction of the plane.
1. **WIND** The vector \( \vec{v} \) represents the speed and direction that the wind is blowing. Suddenly the wind picks up and doubles its speed, but the direction does not change. Write an expression for a vector that describes the new wind velocity in terms of \( \vec{v} \).

2. **SWIMMING** Jan is swimming in a triathlon event. When the ocean water is still, her velocity can be represented by the vector \( \langle 2, 1 \rangle \) miles per hour. During the competition, there was a fierce current represented by the vector \( \langle -1, -1 \rangle \) miles per hour. What vector represents Jan’s velocity during the race?

3. **POLYGONS** Draw a regular polygon around the origin. For each side of the polygon, associate a vector whose magnitude is the length of the corresponding side and whose direction points in the clockwise motion around the origin. What vector represents the sum of all these vectors? Explain.

4. **BASEBALL** Rick is in the middle of a baseball game. His teammate throws him the ball, but throws it far in front of him. He has to run as fast as he can to catch it. As he runs, he knows that as soon as he catches it, he has to throw it as hard as he can to the teammate at home plate. He has no time to stop. In the figure, \( \vec{x} \) is the vector that represents the velocity of the ball *after* Rick throws it and \( \vec{v} \) represents Rick’s velocity because he is running. Assume that Rick can throw just as hard when running as he can when standing still.

   a. What vector would represent the velocity of the ball if Rick threw it the same way but he was standing still?

   b. The angle between \( \vec{x} \) and \( \vec{v} \) is 89°. By running, did it help Rick get the ball to home plate faster than he would have normally been able to if he were standing still?
Dot Product

The dot product of two vectors represents how much the vectors point in the direction of each other. If \( \mathbf{v} \) is a vector represented by \( \langle a, b \rangle \) and \( \mathbf{u} \) is a vector represented by \( \langle c, d \rangle \), the formula to find the dot product is:

\[
\mathbf{v} \cdot \mathbf{u} = ac + bd
\]

Look at the following example:

Graph the vectors and find the dot product of \( \mathbf{v} \) and \( \mathbf{u} \) if

\[
\mathbf{v} = \langle 3, -1 \rangle \text{ and } \mathbf{u} = \langle 2, 5 \rangle.
\]

\[
\mathbf{v} \cdot \mathbf{u} = (3)(2) + (-1)(5) \text{ or } 1
\]

Graph the vectors and find the dot products.

1. \( \mathbf{v} = \langle 2, 1 \rangle \text{ and } \mathbf{u} = \langle -4, 2 \rangle \)
   The dot product is ________

2. \( \mathbf{v} = \langle 3, -2 \rangle \text{ and } \mathbf{u} = \langle 1, 4 \rangle \)
   The dot product is ________

3. \( \mathbf{v} = \langle 0, 3 \rangle \text{ and } \mathbf{u} = \langle 2, 4 \rangle \)
   The dot product is ________

4. \( \mathbf{v} = \langle -1, 4 \rangle \text{ and } \mathbf{u} = \langle -4, 2 \rangle \)
   The dot product is ________

5. Notice the angle formed by the two vectors and the corresponding dot product. Is there any relationship between the type of angle between the two vectors and the sign of the dot product? Make a conjecture.
Student Recording Sheet
Use this recording sheet with pages 610–611 of the Student Edition.

Multiple Choice

Read each question. Then fill in the correct answer.

1. ☐ ☐ ☐ ☐
2. ☐ ☐ ☐ ☐
3. ☐ ☐ ☐ ☐
4. ☐ ☐ ☐ ☐
5. ☐ ☐ ☐ ☐
6. ☐ ☐ ☐ ☐

Short Response/Gridded Response

Record your answer in the blank.

For gridded response questions, also enter your answer in the grid by writing each number or symbol in a box. Then fill in the corresponding circle for that number or symbol.

7. ____________ (grid in)
8. ____________
9. ____________
10. ____________ (grid in)
11. ____________
12. ____________
13. ____________
14. ____________

Extended Response

Record your answers for Question 15 on the back of this paper.
Rubric for Scoring Extended-Response

General Scoring Guidelines

- If a student gives only a correct numerical answer to a problem but does not show how he or she arrived at the answer, the student will be awarded only 1 credit. All extended-response questions require the student to show work.

- A fully correct answer for a multiple-part question requires correct responses for all parts of the question. For example, if a question has three parts, the correct response to one or two parts of the question that required work to be shown is not considered a fully correct response.

- Students who use trial and error to solve a problem must show their method. Merely showing that the answer checks or is correct is not considered a complete response for full credit.

Exercise 15 Rubric

<table>
<thead>
<tr>
<th>Score</th>
<th>Specific Criteria</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>The values of $x = 10$, $y = 16$, and $z = 12.8$ are found using appropriate proportions based upon students knowledge of right triangle geometric mean theorems.</td>
</tr>
<tr>
<td>3</td>
<td>A generally correct solution, but may contain minor flaws in reasoning or computation.</td>
</tr>
<tr>
<td>2</td>
<td>A partially correct interpretation and/or solution to the problem.</td>
</tr>
<tr>
<td>1</td>
<td>A correct solution with no evidence or explanation.</td>
</tr>
<tr>
<td>0</td>
<td>An incorrect solution indicating no mathematical understanding of the concept or task, or no solution is given.</td>
</tr>
</tbody>
</table>
Chapter 8 Quiz 1
(Lessons 8-1 and 8-2)

1. Find the geometric mean between 12 and 16.

For Questions 2 and 3, find \( x \) and \( y \).

2. \[
\begin{array}{c}
\text{x} \\
5 \\
12
\end{array}
\]

3. \[
\begin{array}{c}
y \\
8 \\
9
\end{array}
\]

4. Find \( x \).

5. The measures of the sides of a triangle are 19, 15, and 13. Use the Pythagorean Theorem to classify the triangle as acute, obtuse, or right.

Chapter 8 Quiz 2
(Lessons 8-3 and 8-4)

For Questions 1 and 2, find \( x \).

1. \[
\begin{array}{c}
x \\
6 \\
45^\circ
\end{array}
\]

2. \[
\begin{array}{c}
x \\
6 \\
60^\circ \\
30^\circ
\end{array}
\]

For Questions 3 and 4, find \( x \) to the nearest tenth.

3. \[
\begin{array}{c}
x \\
11 \\
13
\end{array}
\]

4. \[
\begin{array}{c}
x \\
31^\circ \\
17
\end{array}
\]

5. A rectangle has a diagonal 20 inches long that forms angles of \( 60^\circ \) and \( 30^\circ \) with the sides. Find the perimeter of the rectangle.

6. Find \( \sin 52^\circ \). Round to the nearest ten-thousandth.

7. If \( \cos A = 0.8945 \), find \( m\angle A \) to the nearest degree.

8. The distance along a hill is 24 feet. If the land slopes uphill at an angle of \( 8^\circ \), find the vertical distance from the top to the bottom of the hill. Round to the nearest tenth.

9. Use a calculator to find the measure of \( \angle T \) to the nearest tenth.

10. Use a calculator to find the measure of \( \angle T \) to the nearest tenth.
1. Name the angle of elevation in the figure.

2. Find $x$ to the nearest tenth.

3. Solve $\triangle ABC$. Round angle measures to the nearest degree and side measures to the nearest tenth.

4. Solve $\triangle RST$. Round your answers to the nearest degree.

5. A squirrel, 37 feet up in a tree, sees a dog 29 feet from the base of the tree. Find the measure of the angle of depression to the nearest degree.

---

Find the magnitude and direction of each vector.

1. $\overrightarrow{RT}$: $R(14, 6)$ and $T(-5, -10)$

2. $\overrightarrow{PQ}$: $P(20, -16)$ and $Q(-9, -4)$

3. $\overrightarrow{c} + \overrightarrow{d}$

4. $\overrightarrow{k} - \overrightarrow{m}$

5. Lidia is rollerblading south at a velocity of 9 miles per hour. The wind is blowing 3 miles per hour in the opposite direction. What is Lidia’s resultant velocity and direction?
Chapter 8 Mid-Chapter Test
(Lessons 8-1 through 8-4)

Part I Write the letter for the correct answer in the blank at the right of each question.

1. Find the geometric mean between 7 and 9.
   A $3\sqrt{7}$  B 16  C 8  D 2
   1. ___

2. Find $x$.
   F $6\sqrt{6}$  H $6\sqrt{55}$
   G $\frac{\sqrt{2}}{5}$  J $\frac{\sqrt{23}}{5}$
   2. ___

3. Find $\sin C$.
   A $\sqrt{2}$  C $\frac{\sqrt{23}}{\sqrt{2}}$
   B $\frac{\sqrt{2}}{5}$  D $\frac{\sqrt{23}}{5}$
   3. ___

4. Find $x$ to the nearest tenth.
   F 14  H 18.4
   G 21.1  J 32.2
   4. ___

5. Find $y$ to the nearest degree.
   A 145  C 45
   B 60  D 35
   5. ___

Part II

For Questions 6–8, find $x$ and $y$.

6. ___

7. ___

8. ___

9. The measures of the sides of a triangle are 56, 90, and 106. Use the Pythagorean Theorem to classify the triangle as acute, obtuse, or right.

10. Guy wires 80 feet long support a 65-foot tall telephone pole. To the nearest degree, what angle will the wires make with the ground?
Chapter 8 Vocabulary Test

Choose the correct word to complete each sentence.

1. The square root of the product of two numbers is the (geometric mean, or Pythagorean triple) of the numbers.

2. A group of three whole numbers that satisfy the equation \( a^2 + b^2 = c^2 \), where \( c \) is the greatest number, is called a (trigonometric ratio, or Pythagorean triple).

Write whether each sentence is true or false. If false, replace the underlined word or number to make a true sentence.

3. The ratio of the lengths of any two sides of a right triangle is called a geometric mean.

4. An angle between the line of sight and the horizontal when an observer looks upward is called an angle of elevation.

Choose from the terms above to complete each sentence.

5. An angle between the line of sight and the horizontal when an observer looks downward is called a(n) ___?

6. The word ___? ___ is derived from the Greek term for triangle and measure.

7. In a right triangle, the ___? ___ of an angle can be found by dividing the length of the opposite leg by the length of the triangle’s hypotenuse.

8. In a right triangle, the ___? ___ of an angle can be found by dividing the length of the opposite leg by the length of the adjacent leg.

Define each term in your own words.

9. component form

10. Pythagorean Theorem
Write the letter for the correct answer in the blank at the right of each question.

1. Find the geometric mean between 20 and 5.
   A 100  B 50  C 12.5  D 10
   1. ____

2. Find $x$ in $\triangle ABC$.
   F 8  H $\sqrt{20}$
   G 10  J 64
   2. ____

3. Find $x$ in $\triangle PQR$.
   A 13  C 16
   B 15  D $\sqrt{60}$
   3. ____

4. Find $x$ in $\triangle STU$.
   F 2  H $\sqrt{32}$
   G 8  J $\sqrt{514}$
   4. ____

5. Which set of measures could represent the lengths of the sides of a right triangle?
   A 2, 3, 4
   B 7, 11, 14
   C 8, 10, 12
   D 9, 12, 15
   5. ____

6. Find $x$ in $\triangle DEF$.
   F 6  H $6\sqrt{3}$
   G $6\sqrt{2}$  J 12
   6. ____

7. Find $y$ in $\triangle XYZ$.
   A $7.5\sqrt{3}$  C 15
   B $15\sqrt{3}$  D 30
   7. ____

8. The length of the sides of a square is 10 meters. Find the length of the diagonals of the square.
   F 10 m  H $10\sqrt{3}$ m
   G $10\sqrt{2}$ m  J 20 m
   8. ____

9. Find $x$ in $\triangle HJK$.
   A $5\sqrt{2}$  C 10
   B $5\sqrt{3}$  D 15
   9. ____

10. Find $x$ in $\triangle ABC$.
    F 25  H $25\sqrt{3}$
    G $25\sqrt{2}$  J 100
    10. ____
11. Find $x$ to the nearest tenth.
   \[ \begin{array}{cccc}
   \text{A} & 7.3 & \text{C} & 18.4 \\
   \text{B} & 17.3 & \text{D} & 47.1 \\
   \end{array} \]

12. Find the measure of the angle of elevation of the Sun when a pole 25 feet tall casts a shadow 42 feet long.
   \[ \begin{array}{cccc}
   \text{F} & 30.8^\circ & \text{G} & 36.5^\circ \\
   \text{H} & 53.5^\circ & \text{J} & 59.2^\circ \\
   \end{array} \]

13. Which is the angle of depression in the figure at the right?
   \[ \begin{array}{cccc}
   \text{A} & \angle AOT & \text{C} & \angle TOB \\
   \text{B} & \angle AOB & \text{D} & \angle BTO \\
   \end{array} \]

14. Find $y$ in $\triangle XYZ$ if $m\angle Y = 36$, $m\angle X = 49$, and $x = 12$. Round to the nearest hundredth.
   \[ \begin{array}{cccc}
   \text{F} & 0.04 & \text{G} & 9.35 \\
   \text{H} & 14.80 & \text{J} & 15.41 \\
   \end{array} \]

15. To find the distance between two points, $A$ and $B$, on opposite sides of a river, a surveyor measures the distance from $A$ to $C$ as 200 feet, $m\angle A = 72$, and $m\angle B = 37$. Find the distance from $A$ to $B$. Round your answer to the nearest tenth.
   \[ \begin{array}{cccc}
   \text{A} & 77.4 \text{ ft} & \text{B} & 201.2 \text{ ft} \\
   \text{C} & 250.4 \text{ ft} & \text{D} & 314.2 \text{ ft} \\
   \end{array} \]

16. In $\triangle ABC$, $m\angle A = 40$, $m\angle C = 115$, and $b = 8$. Find $a$ to the nearest tenth.
   \[ \begin{array}{cccc}
   \text{F} & 12.2 & \text{G} & 11.3 \\
   \text{H} & 5.7 & \text{J} & 5.3 \\
   \end{array} \]

17. Find the length of the third side of a triangular garden if two sides measure 8 feet and 12 feet and the included angle measures 50.
   \[ \begin{array}{cccc}
   \text{A} & 7.8 \text{ ft} & \text{B} & 9.2 \text{ ft} \\
   \text{C} & 14.4 \text{ ft} & \text{D} & 146.3 \text{ ft} \\
   \end{array} \]

18. In $\triangle DEF$, $d = 20$, $e = 25$, and $f = 30$. Find $m\angle F$ to the nearest degree.
   \[ \begin{array}{cccc}
   \text{F} & 83^\circ & \text{G} & 76^\circ \\
   \text{H} & 56^\circ & \text{J} & 47^\circ \\
   \end{array} \]

19. Find the component form of $\overrightarrow{AB}$ with $A(2, 3)$ and $B(-4, 6)$.
   \[ \begin{array}{cccc}
   \text{A} & \langle -2, 9 \rangle & \text{B} & \langle 2, -9 \rangle \\
   \text{C} & \langle -6, 3 \rangle & \text{D} & \langle 6, -3 \rangle \\
   \end{array} \]

20. Find the magnitude of $\overrightarrow{AB}$ with $A(3, 4)$ and $B(-1, 7)$.
   \[ \begin{array}{cccc}
   \text{F} & \langle 4, -3 \rangle & \text{G} & 5 \\
   \text{H} & \sqrt{13} & \text{J} & 25 \\
   \end{array} \]

**Bonus** In $\triangle ABC$, $a = 50$, $b = 48$, and $c = 40$. Find $m\angle A$ to the nearest degree.
Write the letter for the correct answer in the blank at the right of each question.

1. Find the geometric mean between 7 and 12.
   A 5  B 9.5  C \(\sqrt{19}\)  D \(2\sqrt{21}\)  1. ___

2. In \(\triangle PQR\), \(RS = 4\) and \(QS = 6\). Find \(PS\).
   F 2  G 5  H \(\sqrt{10}\)  J \(2\sqrt{6}\)  2. ___

3. Find \(x\).
   A \(3\sqrt{2}\)  B \(\sqrt{14}\)  C 4.5  D 3  3. ___

4. Find \(y\).
   F 12  G 11  H 9  J 2  4. ___

5. Find the length of the hypotenuse of a right triangle with legs that measure 5 and 7.
   A 12  B \(\sqrt{24}\)  C \(\sqrt{35}\)  D \(\sqrt{74}\)  5. ___

6. Find \(x\).
   F 3  G 4  H \(4\sqrt{3}\)  J \(2\sqrt{5}\)  6. ___

7. Which set of measures could represent the lengths of the sides of a right triangle?
   A 9, 40, 41  B 8, 30, 31  C 7, 8, 15  D \(\sqrt{2}, \sqrt{3}, \sqrt{6}\)  7. ___

8. Find \(c\).
   F 7  G \(7\sqrt{2}\)  H \(7\sqrt{3}\)  J 14  8. ___

9. Find the perimeter of a square if the length of its diagonal is 12 inches. Round to the nearest tenth.
   A 8.5 in.  B 33.9 in.  C 48 in.  D 67.9 in.  9. ___

10. Find \(x\).
    F 4  G \(4\sqrt{2}\)  H \(4\sqrt{3}\)  J \(8\sqrt{3}\)  10. ___
11. Find \( x \) to the nearest tenth.
   \[ \begin{array}{ll}
   \text{A} & 5.8 \\
   \text{B} & 5.9 \\
   \text{C} & 8.1 \\
   \text{D} & 17.3 \\
   \end{array} \]

12. Find \( x \) to the nearest degree.
   \[ \begin{array}{ll}
   \text{F} & 56 \\
   \text{G} & 45 \\
   \text{H} & 34 \\
   \text{J} & 29 \\
   \end{array} \]

13. If a 20-foot ladder makes a 65° angle with the ground, how many feet up a wall will it reach? Round your answer to the nearest tenth.
   \[ \begin{array}{ll}
   \text{A} & 8.5 \text{ ft} \\
   \text{B} & 10 \text{ ft} \\
   \text{C} & 18.1 \text{ ft} \\
   \text{D} & 42.9 \text{ ft} \\
   \end{array} \]

14. A ship’s sonar finds that the angle of depression to a wreck on the bottom of the ocean is 12.5°. If a point on the ocean floor is 60 meters directly below the ship, how many meters is it from that point on the ocean floor to the wreck? Round your answer to the nearest tenth.
   \[ \begin{array}{ll}
   \text{F} & 277.2 \text{ m} \\
   \text{G} & 270.6 \text{ m} \\
   \text{H} & 61.5 \text{ m} \\
   \text{J} & 13.3 \text{ m} \\
   \end{array} \]

15. Find the angle of elevation of the sun if a building 100 feet tall casts a shadow 150 feet long. Round to the nearest degree.
   \[ \begin{array}{ll}
   \text{A} & 60° \\
   \text{B} & 48° \\
   \text{C} & 42° \\
   \text{D} & 34° \\
   \end{array} \]

16. When the Sun’s angle of elevation is 73°, a tree tilted at an angle of 5° from the vertical casts a 20-foot shadow on the ground. Find the length of the tree to the nearest tenth of a foot.
   \[ \begin{array}{ll}
   \text{F} & 6.3 \text{ ft} \\
   \text{G} & 19.2 \text{ ft} \\
   \text{H} & 51.1 \text{ ft} \\
   \text{J} & 219.4 \text{ ft} \\
   \end{array} \]

17. In \( \triangle CDE \), \( m\angle C = 52 \), \( m\angle D = 17 \), and \( e = 28.6 \). Find \( c \) to the nearest tenth.
   \[ \begin{array}{ll}
   \text{A} & 77.1 \\
   \text{B} & 49.1 \\
   \text{C} & 24.1 \\
   \text{D} & 18.4 \\
   \end{array} \]

18. In \( \triangle PQR \), \( p = 56 \), \( r = 17 \), and \( m\angle Q = 110 \). Find \( q \) to the nearest tenth.
   \[ \begin{array}{ll}
   \text{F} & 4076.2 \\
   \text{G} & 63.8 \\
   \text{H} & 52.6 \\
   \text{J} & 3.1 \\
   \end{array} \]

19. Find the component form of \( \overrightarrow{CD} \) with \( C(5, -7) \) and \( D(-3, 9) \).
   \[ \begin{array}{ll}
   \text{A} & (-2, 2) \\
   \text{B} & (2, 2) \\
   \text{C} & (8, -16) \\
   \text{D} & (-8, 16) \\
   \end{array} \]

20. A pilot is flying due east at a speed of 300 miles per hour and wind is blowing due north at 50 miles per hour. What is the magnitude of the resultant velocity of the plane?
   \[ \begin{array}{ll}
   \text{F} & 300 \text{ mph} \\
   \text{G} & 350 \text{ mph} \\
   \text{H} & \text{about 304 mph} \\
   \text{J} & 2500 \text{ mph} \\
   \end{array} \]

Bonus From a window 20 feet above the ground, the angle of elevation to the top of another building is 35°. The distance between the buildings is 52 feet. Find the height of the building to the nearest tenth of a foot.

B: __________________
Write the letter for the correct answer in the blank at the right of each question.

1. Find the geometric mean between 9 and 11.
   A $3\sqrt{11}$  
   B $2\sqrt{5}$  
   C 10  
   D 2  

2. In $\triangle PQR$, $RS = 5$ and $QS = 8$. Find $PS$.
   F 3  
   G 6.5  
   H $\sqrt{13}$  
   J $2\sqrt{10}$  

3. Find $x$.
   A 5.5  
   B $\sqrt{11}$  
   C $\sqrt{24}$  
   D $\sqrt{33}$  

4. Find $y$.
   F 4  
   G 5  
   H 8  
   J 9  

5. Find the length of the hypotenuse of a right triangle whose legs measure 6 and 5.
   A 11  
   B $\sqrt{11}$  
   C $\sqrt{30}$  
   D $\sqrt{61}$  

6. Find $x$.
   F $\sqrt{39}$  
   G 6  
   H $5\sqrt{3}$  
   J 5  

7. Which set of measures could represent the lengths of the sides of a right triangle?
   A $\frac{3}{4}, 1, \frac{5}{4}$  
   B $\sqrt{3}, \sqrt{5}, \sqrt{15}$  
   C 7, 17, 24  
   D 8, 15, 16  

8. Find $c$.
   F 18  
   G $9\sqrt{3}$  
   H $9\sqrt{2}$  
   J 9  

9. Find the perimeter of a square if the length of its diagonal is 16 millimeters. Round to the nearest tenth.
   A 11.3 mm  
   B 45.3 mm  
   C 90.5 mm  
   D 128.0 mm  

10. Find $x$.
    F 6  
    G $6\sqrt{2}$  
    H $6\sqrt{3}$  
    J $12\sqrt{3}$
11. Find $x$.
   A 8.0  C 10.4
   B 8.9  D 10.8

12. Find $x$ to the nearest degree.
   F 57  H 33
   G 55  J 29

13. If a 24-foot ladder makes a $58^\circ$ angle with the ground, how many feet up a wall will it reach? Round your answer to the nearest tenth.
   A 38.4 ft  B 20.8 ft  C 20.4 ft  D 12.7 ft

14. A ship’s sonar finds that the angle of depression to a wreck on the bottom of the ocean is $13.2^\circ$. If a point on the ocean floor is 75 meters directly below the ship, how many meters is it from that point on the ocean floor to the wreck? Round to the nearest tenth.
   F 328.4 m  G 319.8 m  H 77.0 m  J 17.6 m

15. Find the angle of elevation of the sun if a building 125 feet tall casts a shadow 196 feet long. Round to the nearest degree.
   A 64°  B 50°  C 40°  D 33°

16. When the sun’s angle of elevation is $76^\circ$, a tree tilted at an angle of $4^\circ$ from the vertical casts an 18-foot shadow on the ground. Find the length of the tree to the nearest tenth of a foot.
   F 250.4 ft  H 17.7 ft
   G 56.5 ft  J 4.6 ft

17. In $\triangle ABC$, $m\angle A = 46$, $m\angle B = 105$, and $c = 19.8$. Find $a$ to the nearest tenth.
   A 29.4  B 28.5  C 15.7  D 14.7

18. In $\triangle LMN$, $\ell = 42$, $m = 61$, and $m\angle N = 108$. Find $n$ to the nearest tenth.
   F 7068.4  G 84.1  H 79.2  J 24.7

19. Find the component from of $\overrightarrow{EF}$ with $E(-11,-3)$ and $F(7,-4)$
   A $<-18, 1>$  B $<18, -1>$  C $<-4, -7>$  D $<4, 7>$

20. An eagle is traveling along a path due east at a rate of 50 miles per hour and the wind is blowing due north at 15 miles per hour. What is the magnitude of the resultant velocity of the eagle?
   F 35 mph  G 50 mph  H 65 mph  J about 52.2 mph

**Bonus** From a window 24 feet above the ground, the angle of elevation to the top of another building is $38^\circ$. The distance between the buildings is 63 feet. Find the height of the building to the nearest tenth of a foot.

**B:**
1. Find the geometric mean between 2 and 5.

For Questions 2–5, find \( x \).

2. \[
\begin{align*}
2 & \hspace{1cm} x \\
6 & \hspace{1cm} y
\end{align*}
\]

3. \[
\begin{align*}
x & \hspace{1cm} 2 \\
12 &
\end{align*}
\]

4. \[
\begin{align*}
60 & \hspace{1cm} 20 \\
x &
\end{align*}
\]

5. \[
\begin{align*}
20 & \hspace{1cm} x \\
x & \hspace{1cm} 20
\end{align*}
\]

6. Find \( x \).

7. In parallelogram \( ABCD \), \( AD = 4 \) and \( m \angle D = 60 \). Find \( AF \).

8. Find \( x \) and \( y \).

9. Find \( x \) to the nearest tenth.

10. An A-frame house is 40 feet high and 30 feet wide. Find the measure of the angle that the roof makes with the floor. Round to the nearest degree.

11. A 30-foot tree casts a 12-foot shadow. Find the measure of the angle of elevation of the Sun to the nearest degree.
12. A boat is 1000 meters from a cliff. If the angle of depression from the top of the cliff to the boat is 15°, how tall is the cliff? Round your answer to the nearest tenth.

13. A plane flying at an altitude of 10,000 feet begins descending when the end of the runway is 50,000 feet from a point on the ground directly below the plane. Find the measure of the angle of descent (depression) to the nearest degree.

14. Find x to the nearest tenth.

15. Find x to the nearest tenth.

16. A tree grew at a 3° slant from the vertical. At a point 50 feet from the tree, the angle of elevation to the top of the tree is 17°. Find the height of the tree to the nearest tenth of a foot.

17. Find x to the nearest degree.

18. In \( \triangle XYZ \), \( \angle X = 152 \), \( y = 15 \), and \( z = 19 \). Find x to the nearest tenth.

19. Find the magnitude and direction of the vector \( \overrightarrow{AZ} \): A(8, 8) and Z(1, –3).

20. Copy the vectors to find \( \overrightarrow{s} + \overrightarrow{t} \).

**Bonus** Find x.
Chapter 8 Test, Form 2D

1. Find the geometric mean between $3\sqrt{6}$ and $5\sqrt{6}$.

For Questions 2–5, find $x$.

2. 

3. 

4. 

5. 

6. Find $x$.

7. In parallelogram $ABCD$, $AD = 14$ and $m\angle D = 60$. Find $AF$.

8. Find $x$ and $y$.

9. Find $x$ to the nearest tenth.

10. An A-frame house is 45 feet high and 32 feet wide. Find the measure of the angle that the roof makes with the floor. Round to the nearest degree.

11. A 38-foot tree casts a 16-foot shadow. Find the measure of the angle of elevation of the sun to the nearest degree.
12. A boat is 2000 meters from a cliff. If the angle of depression from the top of the cliff to the boat is 10°, how tall is the cliff? Round your answer to the nearest tenth.

13. A plane flying at an altitude of 10,000 feet begins descending when the end of the runway is 60,000 feet from a point on the ground directly below the plane. Find the measure of the angle of descent (depression) to the nearest degree.

14. Find x to the nearest tenth.

15. Find x to the nearest degree.

16. A tree grew at a 3° slant from the vertical. At a point 60 feet from the tree, the angle of elevation to the top of the tree is 14°. Find the height of the tree to the nearest tenth of a foot.

17. Find x to the nearest degree.

18. In \( \triangle XYZ \), \( m \angle X = 156 \), \( y = 18 \), and \( z = 21 \). Find x to the nearest tenth.

19. Find the magnitude and direction of the vector \( \overrightarrow{PQ} \): \( P(-2, 4) \) and \( Q(-5, -6) \).

20. Copy the vectors to find \( \overrightarrow{d} - \overrightarrow{f} \).

**Bonus** Find x.
1. Find the geometric mean between \( \frac{2}{9} \) and \( \frac{3}{9} \).

2. Find \( x \) in \( \triangle PQR \).

3. Find \( x \) in \( \triangle XYZ \).

4. If the length of one leg of a right triangle is three times the length of the other and the hypotenuse is 20, find the length of the shorter leg.

5. Find the measure of the altitude drawn to the hypotenuse of a right triangle with legs that measure 3 and 4.

6. Find \( x \).

7. Richmond is 200 kilometers due east of Teratown and Hamilton is 150 kilometers directly north of Teratown. Find the shortest distance in kilometers between Hamilton and Richmond.

8. The measures of the sides of a triangle are 48, 55, and 73. Use the Pythagorean Theorem to classify the triangle as acute, obtuse, or right.

9. Find the exact perimeter of this square.

10. Find the exact perimeter of rectangle \( ABCD \).

11. Find \( x \) and \( y \).

12. \( \triangle ABC \) is a 30°-60°-90° triangle with right angle \( A \) and with \( AC \) as the longer leg. If \( A(-4, -2) \) and \( B(-4, 6) \), find the coordinates of \( C \).
13. If $\overline{AB} \parallel \overline{CD}$, find $x$ and the exact length of $\overline{CD}$.

14. The angle of elevation from a point on the street to the top of a building is $29^\circ$. The angle of elevation from another point on the street, 50 feet farther away from the building, to the top of the building is $25^\circ$. To the nearest foot, how tall is the building?

15. The angle of depression from the top of a flagpole on top of a lighthouse to a boat on the ocean is $2.9^\circ$. The angle of depression from the bottom of the flagpole to the boat is $2.6^\circ$. If the boat is 400 feet away from shore and the lighthouse is right on the edge of the shore, how tall is the flagpole? Round your answer to the nearest foot.

16. In $\triangle JKL$, $m \angle J = 26.8$, $m \angle K = 19$, and $k = 17$. Find $\ell$ to the nearest tenth.

17. Don hit a golf ball from the tee toward the hole which is 250 yards away. However, due to the wind, his drive was $5^\circ$ off course. If the angle between the segment from the hole to the tee and the segment from the hole to the ball measures $97^\circ$, how far did Don drive the ball? Round to the nearest tenth of a yard.

18. In $\triangle HJK$, $h = 7$, $j = 12.3$, and $k = 7.9$. Find $m \angle K$. Round your answer to the nearest degree.

19. Find the magnitude and direction of the vector $\overrightarrow{WZ}$: $W(15, 25)$ and $Z(10, -6)$.

20. Copy the vectors to find $\overrightarrow{a} + \overrightarrow{b}$ and $\overrightarrow{a} - \overrightarrow{b}$.

**Bonus** A 50-foot vertical pole that stands on a hillside. The slope of the hill side makes an angle of $10^\circ$ with the horizontal. Two guy wires extend from the top of the pole to points on the hill 60 feet uphill and downhill from its base. Find the length of each guy wire to the nearest tenth of a foot.
Demonstrate your knowledge by giving a clear, concise solution to each problem. Be sure to include all relevant drawings and justify your answers. You may show your solution in more than one way or investigate beyond the requirements of the problem.

1. If the geometric mean between 10 and \( x \) is 6, what is \( x \)? Show how you obtained your answer.

2. 

   \[ \begin{align*} 
   x & = \sqrt{8 \cdot 2} \\
   x & = \sqrt{16} \\
   x & = 4 
   \end{align*} \]

   a. Max used the following equations to find \( x \) in \( \triangle PQR \). Is Max correct? Explain.

   b. If \( \angle PRQ \) is a right angle, what is the measure of \( PS \)?

   c. Is \( \triangle PRS \) a 45°-45°-90° triangle? Explain.

3. To solve for \( x \) in a triangle, when would you use \( \sin \) and when would you use \( \sin^{-1} \)? Give an example for each type of situation.

4. Draw a diagram showing the angles of elevation and depression and label each. How are the measures of these angles related?

5. Draw an obtuse triangle and label the vertices, the measures of two angles, and the length of one side. Explain how to solve the triangle.

6. Irina is solving \( \triangle ABC \). She plans to first use the Law of Sines to find two of the angles. Is Irina’s plan a good one? Explain.
1. If \( TA \) bisects \( \angle YTB \), \( TC \) bisects \( \angle BTZ \),
\[
m_\angle YTA = 4y + 6, \quad \text{and} \quad m_\angle BTC = 7y - 4,
\]
find \( m_\angle CTZ \). (Lesson 1-4)

A 52  
B 38  
C 25  
D 8

2. Which statement is always true? (Lesson 2-5)

F If right triangle \( QPR \) has sides \( q, p, \) and \( r \), where \( r \) is the hypotenuse,
\[
then \quad r^2 = p^2 + q^2.
\]

G If \( EF \parallel HJ \), then \( EF = HJ \).

H If \( KL \) and \( VT \) are cut by a transversal, then \( KL \parallel VT \).

J If \( DR \) and \( RH \) are congruent, then \( R \) bisects \( DH \).

3. The equation for \( PT \) is \( y - 2 = 8(x + 3) \). Determine an equation for a
line perpendicular to \( PT \). (Lesson 3-4)

A \( y = \frac{1}{8} x - 7 \)  
B \( y = 8x - 13 \)  
C \( y = -\frac{1}{8} x + 2 \)  
D \( y = -8x \)

4. Angle \( Y \) in \( \triangle XYZ \) measures 90°. \( \overline{XY} \) and \( \overline{YZ} \) each measure 16 meters.
Classify \( \triangle XYZ \). (Lesson 4-1)

F acute and isosceles  
H right and scalene  
G equiangular and equilateral  
J right and isosceles

5. Two sides of a triangle measure 4 inches and 9 inches. Determine which cannot be the perimeter of the triangle. (Lesson 5-3)

A 19 in.  
B 21 in.  
C 23 in.  
D 26 in.

6. \( \triangle ABC \sim \triangle STR \), so \( \frac{AB}{CA} = \frac{?}{?} \). (Lesson 6-2)

F \( \frac{AB}{BC} \)  
G \( \frac{ST}{RS} \)  
H \( \frac{TR}{RS} \)  
J \( \frac{RS}{ST} \)

7. The Petronas Towers in Kuala Lumpur, Malaysia, are 452 meters tall. A woman who is 1.75 meters tall stands 120 meters from the base of one tower. Find the angle of elevation between the woman’s hat and the top of the tower. Round to the nearest tenth. (Lesson 8-5)

A 14.8°  
B 34.9°  
C 55°  
D 75.1°

8. Which equation can be used to find \( x \)? (Lesson 8-4)

F \( x = y \sin 73° \)  
H \( x = \frac{y}{\cos 73°} \)  
G \( x = y \cos 73° \)  
J \( x = \frac{y}{\sin 73°} \)

9. Which inequality describes the possible values of \( x \)? (Lesson 5-6)

A \( x < 4 \)  
B \( x < 2 \)  
C \( x > 2 \)  
D \( x > 4 \)
10. Ashley wants to make a poster for her campaign from a photograph. She uses a photocopier to enlarge the 4 inch by 6 inch photograph. What are the dimensions of the poster if she increases the size of the photograph by a scale factor of 5? (Lesson 7-7)

F 2000 inches by 3000 inches  
H 9 inches by 11 inches  
G 0.8 inches by 1.2 inches  
J 20 inches by 30 inches  

11. Find \( x \). (Lesson 6-2)

A \(-3\)  
B \(3\)  
C \(5\)  
D \(9\)  

12. Trapezoid \(ABCD\) has vertices \(A(1, 6), B(-2, 6), C(-10, -10),\) and \(D(20, -10)\). Find the measure of \(ABCD\)'s midsegment to the nearest tenth. (Lesson 6-6)

F \(3\)  
G \(5.3\)  
H \(7.2\)  
J \(16.5\)  

13. Find \(QP\) to the nearest tenth. (Lesson 8-2)

A \(7.5\)  
B \(12\)  
C \(18.3\)  
D \(19.6\)  

14. Find \(LM\). (Lesson 8-3)

F \(5\)  
G \(5\sqrt{3}\)  
H \(9\)  
J \(10\sqrt{3}\)  

15. Find \(x\) so that \(\ell \parallel m\). (Lesson 3-5)

16. Find \(c\) to the nearest tenth. (Lesson 8-6)
For Questions 17 and 18, complete the following proof. (Lesson 2-7)

Given: $\overline{JK} \cong \overline{LM}$
$\overline{HJ} \cong \overline{KL}$

Prove: $\overline{HK} \cong \overline{KM}$

Proof:

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $\overline{JK} \cong \overline{LM}$, $\overline{HJ} \cong \overline{KL}$</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. $\overline{JK} = \overline{LM}$, $\overline{HJ} = \overline{KL}$</td>
<td>2. (Question 17)</td>
</tr>
<tr>
<td>4. $\overline{HJ} + \overline{JK} = \overline{KL} + \overline{LM}$</td>
<td>4. Add. Prop. of Equality</td>
</tr>
<tr>
<td>5. $\overline{HK} = \overline{KM}$</td>
<td>5. Substitution Prop.</td>
</tr>
<tr>
<td>6. $\overline{HK} \cong \overline{KM}$</td>
<td>6. Def. of $\cong$ segments</td>
</tr>
</tbody>
</table>

For Questions 19 and 20, use the figure at the right.

19. Find the measure of the numbered angles if $m\angle ABC = 57$ and $m\angle BCE = 98$. (Lesson 4-2)

20. If $\overline{BD}$ is a midsegment of $\triangle ABC$, $AD = 2x - 6$, and $DC = 22.5 - 4x$, find $AC$. (Lesson 5-2)

21. If $\triangle DEF \cong \triangle HJK$, $m\angle D = 26$, $m\angle J = 3x + 5$, and $m\angle F = 92$, find $x$. (Lesson 4-3)

22. Use the Exterior Angle Inequality Theorem to list all of the angles with measures that are less than $m\angle 1$. (Lesson 5-3)

23. a. Determine whether $\triangle EFH \sim \triangle JGH$. Justify your answer. (Lesson 7-3)

b. If $G$ is the midpoint of $\overline{FH}$, find $x$. (Lesson 8-3)

c. Use the value of $x$ you found in part b to find the scale factor of $\triangle EFH$ to $\triangle JGH$. (Lesson 7-3)
8-1 Study Guide and Intervention

Geometric Mean

The geometric mean between two numbers is the positive square root of their product. For two positive numbers \( a \) and \( b \), the geometric mean of \( a \) and \( b \) is \( \sqrt{ab} \).

Example

Find the geometric mean between each pair of numbers.

a. 12 and 3

\[ x = \sqrt{12 \cdot 3} \]

\[ x = 6 \]

The geometric mean between 12 and 3 is 6.

b. 8 and 4

\[ x = \sqrt{8 \cdot 4} \]

\[ x = 4 \]

The geometric mean between 8 and 4 is 4 or about 5.7.

Exercises

Find the geometric mean between each pair of numbers.

1. 4 and 4

2. 4 and 6

3. 6 and 9

4. 1.2 and 2.1

5. 12 and 20

6. 4 and 25

7. 16 and 30

8. 10 and 100

9. \( \frac{1}{2} \) and \( \frac{1}{4} \)

10. 10.17 and 3

11. 4 and 16

12. 3 and 24

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3. Study Guide and Intervention (continued)

Geometric Mean

Geometric Means in Right Triangles

In the diagram, \( \triangle ADC \sim \triangle BDC \). An altitude to the hypotenuse of a right triangle forms two right triangles. The two triangles are similar and each is similar to the original triangle.

Example 1

Find the geometric mean between each pair of numbers.

1. \( 4 \text{ and } 7 \)

2. \( 28 \text{ and } 14 \)

3. \( 7 \text{ and } 36 \)

Example 2

Find \( x \), \( y \), and \( z \).

\[
15 = \sqrt{RP \cdot SP} \quad \text{(Geometric Mean, leg theorem)}
\]

\[
15 = \sqrt{25x} \quad \text{Square each side.}
\]

\[
x = 6 \quad \text{Multiply.}
\]

\[
y = 10 \quad \text{Simplify.}
\]

\[
z = 15 \quad \text{Simplify.}
\]

Exercises

Find \( x \), \( y \), and \( z \) to the nearest tenth.

1. \( x = \sqrt{3} \approx 1.7 \)

2. \( x = \sqrt{10} \approx 3.2 \); \( y = \sqrt{14} \approx 3.7 \); \( z = \sqrt{35} \approx 5.9 \)

3. \( x = 3 \); \( y = \sqrt{72} \text{ or } 6\sqrt{2} \approx 8.5 \); \( z = \sqrt{8} \text{ or } 2\sqrt{2} \approx 2.8 \)

4. \( x = \sqrt{15} \approx 3.9 \)

5. \( x = 2 \); \( y = \sqrt{8} \text{ or } 2\sqrt{2} \approx 2.8 \); \( z = \sqrt{8} \text{ or } 2\sqrt{2} \approx 2.8 \)

6. \( x = \sqrt{12} \text{ or } 2\sqrt{3} \approx 3.5 \); \( y = \sqrt{8} \text{ or } 2\sqrt{2} \approx 2.8 \); \( z = \sqrt{24} \text{ or } 2\sqrt{6} \approx 4.9 \)

Chapter 8

Glencoe Geometry
8-1 Practice

**Geometric Mean**

Find the geometric mean between each pair of numbers.

1. 8 and 12
   \[ \sqrt{96} = 9.8 \] or \[ 4\sqrt{6} \approx 9.8 \]
2. 3 and 15
   \[ \sqrt{45} = 6.7 \] or \[ 3\sqrt{5} \approx 6.7 \]
3. \( \frac{5}{2} \) and 2
   \[ \sqrt{\frac{5}{4}} = 1.3 \] or \[ \frac{2\sqrt{10}}{5} \approx 1.3 \]

Write a similarity statement identifying the three similar triangles in the figure.

4. \( \triangle VUT \sim \triangle UAT \sim \triangle VAU \)
5. \( \triangle JLK \sim \triangle LMK \sim \triangle JML \)

Find \( x \), \( y \), and \( z \).

6. \( x = \sqrt{184} \) or \( 2\sqrt{46} \approx 13.6; \)
   \( y = \sqrt{248} \) or \( 2\sqrt{62} \approx 15.7; \)
   \( z = \sqrt{713} \approx 26.7 \)

7. \( x = \sqrt{114} \approx 10.7; \)
   \( y = \sqrt{150} \) or \( 5\sqrt{6} \approx 12.2; \)
   \( z = \sqrt{475} \) or \( 5\sqrt{19} \approx 21.8 \)

8. \( x = 4.5; \)
   \( x = \sqrt{13} \approx 3.6; 6.5 \)

9. \( x = 15; \)
   \( y = 5; \)
   \( z = \sqrt{300} \) or \( 10\sqrt{3} \approx 17.3 \)

10. **Civil** An airport, a factory, and a shopping center are at the vertices of a right triangle formed by three highways. The airport and factory are 6.0 miles apart. Their distances from the shopping center are 3.6 miles and 4.8 miles, respectively. A service road will be constructed from the shopping center to the highway that connects the airport and factory. What is the shortest possible length for the service road? Round to the nearest hundredth. \( 2.88 \) mi

8-1 Word Problem Practice

**Geometric Mean**

1. **Squares** Wilma has a rectangle of dimensions \( \ell \) by \( w \). She would like to replace it with a square that has the same area. What is the side length of the square with the same area as Wilma's rectangle?
   \[ \sqrt{\ell w} \]
2. **Equality** Gretchen computed the geometric mean of two numbers. One of the numbers was 7 and the geometric mean turned out to be 7 as well. What was the other number?
   \[ 7 \]
3. **Viewing Angle** A photographer wants to take a picture of a beach front. His camera has a viewing angle of 90° and he wants to make sure two palm trees located at points \( A \) and \( B \) in the figure are just inside the edges of the photograph.
   He walks out on a walkway that goes over the ocean to get the shot. If his camera has a viewing angle of 90°, at what distance down the walkway should he stop to take his photograph?
   \[ 60 \text{ ft} \]
4. **Exhibitions** A museum has a famous statue on display. The curator places the statue in the corner of a rectangular room and builds a 15-foot-long railing in front of the statue. Use the information below to find how close visitors will be able to get to the statue.
   \[ 7.2 \text{ ft} \]
5. **Cliffs** A bridge connects to a tunnel as shown in the figure. The bridge is 180 feet above the ground. At a distance of 235 feet along the bridge out of the tunnel, the angle to the base and summit of the cliff is a right angle.
   \[ a. \] What is the height of the cliff? Round to the nearest whole number.
   \[ 307 \text{ ft} \]
   \[ b. \] How high is the cliff from base to summit? Round to the nearest whole number.
   \[ 487 \text{ ft} \]
   \[ c. \] What is the value of \( d \)? Round to the nearest whole number.
   \[ 387 \text{ ft} \]
Chapter 8

8-1 Enrichment

Mathematics and Music

Pythagoras, a Greek philosopher who lived during the sixth century B.C., believed that all nature, beauty, and harmony could be expressed by whole-number relationships. Most people remember Pythagoras for his teachings about right triangles. (The sum of the squares of the legs equals the square of the hypotenuse.) But Pythagoras also discovered relationships between the musical notes of a scale. These relationships can be expressed as ratios.

When you play a stringed instrument, you produce different notes by placing your finger on different places on a string. This is the result of changing the length of the vibrating part of the string.

Suppose a C string has a length of 16 inches. Write and solve proportions to determine what length of string would have to vibrate to produce the remaining notes of the scale.

1. D 14 2/3 in. 2. E 12 4/5 in. 3. F 12 in.
7. C 8 in.

8. Complete to show the distance between finger positions on the 16-inch C string for each note. For example, C(16) − D(14 2/3) = 17 9/10


8-2 Study Guide and Intervention

The Pythagorean Theorem and Its Converse

The Pythagorean Theorem

In a right triangle, the sum of the squares of the lengths of the legs equals the square of the length of the hypotenuse. If the three whole numbers a, b, and c satisfy the equation

\[ a^2 + b^2 = c^2 \]

then the numbers a, b, and c form a Pythagorean triple.

Example

a. Find a.

b. Find c.

Find x.

1. \[ \sqrt{18} \] or \[ 3\sqrt{2} \] = 4.2

2. \[ \sqrt{9} \] or \[ 3 \] = 11

3. \[ \sqrt{45} \] or \[ 3\sqrt{5} \] = 6.0

4. \[ \sqrt{1} \] or \[ 1 \] = 1

5. \[ \sqrt{1345} \] = 47

6. \[ \sqrt{663} \] = 25.7

Use a Pythagorean Triple to find x.

7. \[ \sqrt{15^2 + 18^2} \] = 24

8. \[ \sqrt{51^2 + 8^2} \] = 51

9. \[ \sqrt{100^2 + 26^2} \] = 100
**Converse of the Pythagorean Theorem** If the sum of the squares of the lengths of the two shorter sides of a triangle equals the square of the length of the longest side, then the triangle is a right triangle.

You can also use the lengths of sides to classify a triangle:

- If \( a^2 + b^2 = c^2 \), then \( \triangle ABC \) is a right triangle.
- If \( a^2 + b^2 > c^2 \), then \( \triangle ABC \) is acute.
- If \( a^2 + b^2 < c^2 \), then \( \triangle ABC \) is obtuse.

**Example** Determine whether \( \triangle PQR \) is a right triangle.

\[
\begin{align*}
10^2 + (10\sqrt{3})^2 & = 20^2 \quad \text{Compare } c^2 \text{ and } a^2 + b^2 \\
100 + 300 & = 400 \quad \text{Simplify}.
\end{align*}
\]

Since \( c^2 = a^2 + b^2 \), the triangle is a right triangle.

**Exercises**

Determine whether each set of measures can be the measures of the sides of a triangle. If so, classify the triangle as **acute**, **obtuse**, or **right**. Justify your answer.

1. 30, 40, 50
2. 20, 30, 40
3. 18, 24, 30

   - yes, right;  
   - yes, obtuse;  
   - yes, right;

4. 6, 8, 9
5. 6, 12, 18
6. 10, 15, 20

   - yes, acute;  
   - no; 6 + 12 = 18  
   - yes, obtuse;  
   - yes, obtuse;  

7. \( \sqrt{5}, \sqrt{12}, \sqrt{13} \)
8. 2, \( \sqrt{8}, \sqrt{12} \)
9. 9, 40, 41

   - yes, acute;  
   - yes, right;  
   - yes, right;

10. \( \sqrt{13}^2 + (\sqrt{5})^2 = (\sqrt{12})^2 \)
11. \( \sqrt{8}^2 + 2^2 = 41^2 = 40^2 + 9^2 \)

Determine whether each set of numbers can be measure of the sides of a triangle. If so, classify the triangle as **acute**, **obtuse**, or **right**. Justify your answer.

13. 7, 24, 25
14. 8, 14, 20
15. 12.5, 13, 26

   - Yes, right triangle  
   - Yes, obtuse triangle  
   - No, 12.5 + 13 < 26

16. 3\( \sqrt{2} \), \( \sqrt{7} \), 4
17. 20, 21, 29
18. 32, 35, 70

   - Yes, acute triangle  
   - Yes, right triangle  
   - No, 32 + 35 < 70

\[
\begin{align*}
(3\sqrt{2})^2 + \sqrt{7}^2 & > 4^2 \\
20^2 + 21^2 & = 29^2
\end{align*}
\]
8-2 Practice

The Pythagorean Theorem and Its Converse

Find x.

1. \[ \sqrt{698} \approx 26.4 \]

2. \[ \sqrt{715} \approx 26.7 \]

3. \[ \sqrt{595} \approx 24.4 \]

4. \[ \sqrt{1640} \approx 40.5 \]

Use a Pythagorean Triple to find x.

7. \[ \sqrt{27} \]

8. \[ \sqrt{45} \]

9. \[ \sqrt{52} \]

10. \[ \sqrt{64} \]

Determine whether each set of numbers can be measure of the sides of a triangle. If so, classify the triangle as acute, obtuse, or right. Justify your answer.

11. 10, 11, 20

12. 12, 14, 49

13. 5\sqrt{2}, 10, 11

14. 21.5, 24, 55.5

15. 30, 40, 50

16. 65, 72, 97

17. CONSTRUCTION The bottom end of a ramp at a warehouse is 10 feet from the base of the main dock and is 11 feet long. How high is the dock? about 4.6 ft high

18. RIGHT ANGLES Clyde makes a triangle using three sticks of lengths 20 inches, 21 inches, and 28 inches. Is the triangle a right triangle? Explain.

b. Complete the following table.

<table>
<thead>
<tr>
<th>m, n</th>
<th>a</th>
<th>b</th>
<th>c</th>
</tr>
</thead>
<tbody>
<tr>
<td>2, 1</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>3, 2</td>
<td>10</td>
<td>12</td>
<td>13</td>
</tr>
<tr>
<td>4, 1</td>
<td>15</td>
<td>8</td>
<td>17</td>
</tr>
<tr>
<td>5, 2</td>
<td>14</td>
<td>16</td>
<td>20</td>
</tr>
<tr>
<td>5, 3</td>
<td>7</td>
<td>24</td>
<td>25</td>
</tr>
<tr>
<td>7, 1</td>
<td>24</td>
<td>10</td>
<td>26</td>
</tr>
</tbody>
</table>

c. Find a Pythagorean triple that corresponds to a right triangle with a hypotenuse 251 = 625 miles long. (Hint: Use the table you completed for Exercise b to find two positive integers m and n with m > n and \( m^2 + n^2 = 625 \).)

Sample answer: Take \( m = 24 \) and \( n = 7 \) to get \( a = 527 \), \( b = 336 \), and \( c = 625 \).
8-2 Enrichment

Converse of a Right Triangle Theorem

You have learned that the measure of the altitude from the vertex of the right angle of a right triangle to its hypotenuse is the geometric mean between the measures of the two segments of the hypotenuse. Is the converse of this theorem true? In order to find out, it will help to rewrite the original theorem in if-then form as follows.

If \( \triangle ABQ \) is a right triangle with right angle at \( Q \), then \( QU \) is the geometric mean between \( AP \) and \( PB \), where \( P \) is between \( A \) and \( B \) and \( QU \) is perpendicular to \( AB \).

1. Write the converse of the if-then form of the theorem.

If \( QU \) is the geometric mean between \( AP \) and \( PB \), where \( P \) is between \( A \) and \( B \) and \( QU \) is perpendicular to \( AB \), then \( \triangle ABQ \) is a right triangle with right angle at \( Q \).

2. Is the converse of the original theorem true? Refer to the figure at the right to explain your answer.

Yes; \((PQ)^2 = (AP)(PB)\) implies that \(\frac{PQ}{AP} = \frac{PB}{PQ}\).

Since both \( \angle APQ \) and \( \angle QPB \) are right angles, they are congruent. Therefore \( \triangle APQ \sim \triangle QPB \) by SAS similarity. So \( \angle A \equiv \angle QPB \) and \( \angle AQP \equiv \angle B \). But the acute angles of \( \triangle AQP \) are complementary and \( m\angle AQB = m\angle AQP + m\angle PQB \). Hence \( m\angle AQB = 90 \) and \( \triangle AQB \) is a right triangle with right angle at \( Q \).

You may find it interesting to examine the other theorems in Chapter 8 to see whether their converses are true or false. You will need to restate the theorems carefully in order to write their converses.

8-2 Spreadsheet Activity

Pythagorean Triples

You can use a spreadsheet to determine whether three whole numbers form a Pythagorean triple.

Example 1 Use a spreadsheet to determine whether the numbers 12, 16, and 20 form a Pythagorean triple.

Step 1 In cell A1, enter 12. In cell B1, enter 16 and in cell C1, enter 20. The longest side should be entered in column C.

Step 2 In cell D1, enter an equals sign followed by IF(A1^2+B1^2=C1^2,"YES","NO"). This will return "YES" if the set of numbers is a Pythagorean triple and will return "NO" if it is not.

The numbers 12, 16, and 20 form a Pythagorean triple.

Example 2 Use a spreadsheet to determine whether the numbers 3, 6, and 12 form a Pythagorean triple.

Step 1 In cell A2, enter 3, in cell B2, enter 6, and in cell C2, enter 12.

Step 2 Click on the bottom right corner of cell D1 and drag it to D2. This will determine whether or not the set of numbers is a Pythagorean triple.

The numbers 3, 6, and 12 do not form a Pythagorean triple.

Exercises

Use a spreadsheet to determine whether each set of numbers forms a Pythagorean triple.

1. 14, 48, 50
   - yes
   - yes

2. 16, 30, 34
   - yes
   - no

3. 5, 5, 9
   - yes
   - no

4. 4, 5, 7
   - yes
   - yes

5. 8, 2, 4, 5
   - yes
   - no

6. 6, 10, 24, 26
   - yes
   - yes

7. 7, 25, 60, 65
   - yes
   - no

8. 11, 5, 12, 13
   - yes
   - yes

9. 12, 20, 48, 52
   - yes
   - yes
8-3 Study Guide and Intervention

**Special Right Triangles**

**Properties of 45°-45°-90° Triangles**

The sides of a 45°-45°-90° right triangle have a special relationship.

**Example 1**

If the leg of a 45°-45°-90° right triangle is \( x \) units, then the hypotenuse is \( \sqrt{2} \) times the leg. If the hypotenuse is 6 units, find the length of each leg.

Using the Pythagorean Theorem with \( a = b = x \), then

\[
\begin{align*}
    c^2 &= a^2 + b^2 \\
    c^2 &= x^2 + x^2 \\
    c^2 &= 2x^2 \\
    c &= \sqrt{2}x \\
\end{align*}
\]

Exercises

**Find** \( x \).

1. \( \sqrt{2} \approx 11.3 \)
2. \( \sqrt{3} \approx 5.7 \)
3. \( \sqrt{4} \approx 12.7 \)
4. \( \sqrt{8} \approx 11.3 \)

**Answers**

1. \( x = 1 \); \( y = 0.5\sqrt{3} \approx 0.9 \)
2. \( x = 8\sqrt{3} \approx 13.9 \);
   \( y = 16 \)
3. \( x = 5.5 \);
   \( y = 5.5\sqrt{3} \approx 9.5 \)
4. \( x = 9 \);
   \( y = 4\sqrt{3} \approx 6.9 \)
5. \( x = 8 \);
   \( y = 8\sqrt{3} \approx 13.9 \)
6. \( x = 10\sqrt{3} \approx 17.3 \);
   \( y = 10 \)

8. Find the length of the side of an equilateral triangle that has an altitude length of 45 centimeters.

**Properties of 30°-60°-90° Triangles**

The sides of a 30°-60°-90° right triangle also have a special relationship.

**Example 1**

In a 30°-60°-90° right triangle the hypotenuse is twice the shorter leg. Show that the longer leg is \( \sqrt{3} \) times the shorter leg.

\( \triangle MNQ \) is a 30°-60°-90° right triangle, and the length of the hypotenuse \( MN \) is two times the length of the shorter side \( NQ \).

Use the Pythagorean Theorem.

\[
\begin{align*}
    a^2 &= (2x)^2 - x^2 \\
    a^2 &= 4x^2 - x^2 \\
    a^2 &= 3x^2 \\
    a &= \sqrt{3}x \\
\end{align*}
\]

**Example 2**

In a 30°-60°-90° right triangle, the hypotenuse is 5 centimeters. Find the lengths of the other two sides of the triangle.

If the hypotenuse of a 30°-60°-90° right triangle is 5 centimeters, then the length of the shorter leg is one-half of 5, or 2.5 centimeters. The length of the longer leg is \( \sqrt{3} \) times the shorter leg, or (2.5)(\( \sqrt{3} \)) centimeters.

Exercises

**Find** \( x \) and \( y \).

1. \( \sqrt{2} \approx 11.3 \)
2. \( \sqrt{3} \approx 5.7 \)
3. \( \sqrt{4} \approx 12.7 \)
4. \( \sqrt{8} \approx 11.3 \)
5. \( x = 8\sqrt{3} \approx 13.9 \);
   \( y = 16 \)
6. \( x = 5.5 \);
   \( y = 5.5\sqrt{3} \approx 9.5 \)
7. \( x = 9 \);
   \( y = 4\sqrt{3} \approx 6.9 \)
8. \( x = 8 \);
   \( y = 8\sqrt{3} \approx 13.9 \)
9. \( x = 10\sqrt{3} \approx 17.3 \);
   \( y = 10 \)
8-3 Skills Practice

Special Right Triangles

Find $x$.

1. \(45^\circ\) \(25\)
2. \(45^\circ\) \(17\)
3. \(45^\circ\) \(48\)
4. \(45^\circ\) \(100\)
5. \(45^\circ\) \(88\)

7. Determine the length of the leg of a 45°-45°-90° triangle with a hypotenuse length of \(38\).

8. Find the length of the hypotenuse of a 45°-45°-90° triangle with a leg length of 50 centimeters.

Find $x$ and $y$.

9. \(30^\circ\) \(11\)
10. \(30^\circ\) \(8\)
11. \(30^\circ\) \(5\)

12. \(11\sqrt{3}\)
13. \(4\sqrt{3}\)
14. \(10\sqrt{3}\)

15. An equilateral triangle has an altitude length of 27 feet. Determine the length of a side of the triangle.

16. Find the length of the side of an equilateral triangle that has an altitude length of 11\(\sqrt{3}\) feet.

17. \(22\); \(11\sqrt{3}\)
18. \(12\); \(4\sqrt{3}\)
19. \(15\); \(15\sqrt{3}\)

20. \(42\); \(21\)
21. \(78\); \(26\sqrt{3}\)

15. An equilateral triangle has an altitude length of 27 feet. Determine the length of a side of the triangle.

16. Find the length of the side of an equilateral triangle that has an altitude length of 11\(\sqrt{3}\) feet.

17. \(18\sqrt{3}\)
18. \(22\sqrt{3}\)
19. \(26\sqrt{3}\)

14. BOTANICAL GARDENS One of the displays at a botanical garden is an herb garden planted in the shape of a square. The square measures 6 yards on each side. Visitors can view the herbs from a diagonal pathway through the garden. How long is the pathway?

6\(\frac{1}{2}\) yd or about 8.49 yd
**Word Problem Practice**

1. **ORIGAMI** A square piece of paper 150 millimeters on a side is folded in half along a diagonal. The result is a 45°-45°-90° triangle. What is the length of the hypotenuse of this triangle?

\[ 150 \sqrt{2} \text{ mm} \]

2. **ESCALATORS** A 40-foot-long escalator rises from the first floor to the second floor of a shopping mall. The escalator makes a 30° angle with the horizontal.

\[ \frac{4 \sqrt{3}}{3} \text{ m} \approx 2.31 \text{ m} \]

3. **HEXAGONS** A box of chocolates shaped like a regular hexagon is placed snugly inside of a rectangular box as shown in the figure.

If the side length of the hexagon is 3 inches, what are the dimensions of the rectangular box?

6 in. by 3\(\sqrt{3} \) in.

4. **WINDOWS** A large stained glass window is constructed from six 30°-60°-90° triangles as shown in the figure.

The angle that Kim’s line of sight to the top of the screen makes with the horizontal is 30°. The angle that Yolanda’s line of sight to the top of the screen makes with the horizontal is 45°.

- **a.** How high is the top of the screen in terms of \(x\)?
  \[ x \text{ ft} \]

- **b.** What is \(x + \frac{15}{x}\)?
  \[ \sqrt{3} \text{ ft} \]

- **c.** How far is Yolanda from the screen? Round your answer to the nearest tenth.
  \[ 20.5 \text{ ft} \]

**Enrichment**

**Constructing Values of Square Roots**

The diagram at the right shows a right isosceles triangle with two legs of length 1 inch. By the Pythagorean Theorem, the length of the hypotenuse is \(\sqrt{2}\) inches. By constructing an adjacent right triangle with legs of \(\sqrt{2}\) inches and 1 inch, you can create a segment of length \(\sqrt{3}\).

By continuing this process as shown below, you can construct a “wheel” of square roots. This wheel is called the “Wheel of Theodorus” after a Greek philosopher who lived about 400 B.C.

Continue constructing the wheel until you make a segment of length \(\sqrt{18}\).
**Trigonometric Ratios**

The ratio of the lengths of two sides of a right triangle is called a **trigonometric ratio**. The three most common ratios are sine, cosine, and tangent, which are abbreviated sin, cos, and tan, respectively.

\[
\sin R = \frac{\text{opposite}}{\text{hypotenuse}} \quad \cos R = \frac{\text{adjacent}}{\text{hypotenuse}} \quad \tan R = \frac{\text{opposite}}{\text{adjacent}}
\]

**Example**

Find \( \sin A, \cos A, \) and \( \tan A \). Express each ratio as a fraction and a decimal to the nearest hundredth.

\[
\sin A = \frac{BC}{BA} = \frac{5}{13} \approx 0.38 \\
\cos A = \frac{AC}{AB} = \frac{12}{13} \approx 0.92 \\
\tan A = \frac{BC}{AC} = \frac{5}{12} \approx 0.42
\]

**Exercises**

Find \( \sin J, \cos J, \tan J, \sin L, \cos L, \) and \( \tan L \). Express each ratio as a fraction and as a decimal to the nearest hundredth if necessary.

1. \( \sin J = \frac{12}{20} = 0.6; \quad \cos J = \frac{16}{20} = 0.8; \quad \tan J = \frac{16}{12} = 1.33 \)

2. \( \sin J = \frac{36}{40} = 0.6; \quad \cos J = \frac{24}{40} = 0.8; \quad \tan J = \frac{24}{32} = 0.75 \)

3. \( \sin J = \frac{36}{24} = 0.87; \quad \cos J = \frac{24\sqrt{3}}{24} = 0.5; \quad \tan J = \frac{24\sqrt{3}}{36} = 1.33 \)

4. \( \sin L = \frac{12}{20} = 0.6; \quad \cos L = \frac{16}{20} = 0.8; \quad \tan L = \frac{12\sqrt{3}}{16} = 0.58 \)

5. \( \sin L = \frac{24}{36} = 0.6; \quad \cos L = \frac{24}{36} = 0.6; \quad \tan L = \frac{12\sqrt{3}}{36} = 0.58 \)

6. \( \sin L = \frac{24}{36} = 0.6; \quad \cos L = \frac{24}{36} = 0.6; \quad \tan L = \frac{12\sqrt{3}}{36} = 0.58 \)

Use Inverse Trigonometric Ratios

You can use a calculator and the sine, cosine, or tangent to find the measure of the angle, called the inverse of the trigonometric ratio.

**Example**

Use a calculator to find the measure of \( \angle T \) to the nearest tenth.

The measures given are those of the leg opposite \( \angle T \) and the hypotenuse, so write an equation using the sine ratio.

\[
\sin T = \frac{\text{opposite}}{\text{hypotenuse}}
\]

If \( \sin T = \frac{29}{34} \), then \( m \angle T = 71.5^\circ \).

Use a calculator. So, \( m \angle T = 71.5^\circ \).

**Exercises**

Use a calculator to find the measure of \( \angle T \) to the nearest tenth.

1. \( 35.5^\circ \)

2. \( 22.9^\circ \)

3. \( 67^\circ \)

4. \( 71.5^\circ \)

5. \( 75^\circ \)

6. \( 30.5^\circ \)
8-4 Skills Practice

Trigonometry

Find \( \sin R, \cos R, \tan R, \sin S, \cos S, \text{ and } \tan S. \)
Express each ratio as a fraction and as a decimal to the nearest hundredth.

1. \( r = 16, \ s = 30, \ t = 34 \)
   \[ \sin R = \frac{8}{17} \approx 0.47; \quad \cos R = \frac{15}{17} \approx 0.88; \quad \tan R = \frac{12}{17} \approx 0.71 \]
   \[ \sin S = \frac{8}{15} \approx 0.53; \quad \cos S = \frac{15}{17} \approx 0.88; \quad \tan S = \frac{12}{15} = 0.80 \]

2. \( r = 10, \ s = 24, \ t = 26 \)
   \[ \sin R = \frac{5}{13} \approx 0.38; \quad \cos R = \frac{12}{13} \approx 0.92; \quad \tan R = \frac{12}{5} = 2.4 \]
   \[ \sin S = \frac{12}{13} \approx 0.92; \quad \cos S = \frac{5}{13} \approx 0.38; \quad \tan S = \frac{5}{12} = 0.42 \]

Use a special right triangle to express each trigonometric ratio as a fraction and as a decimal to the nearest hundredth if necessary.

3. \( \sin 30^\circ = \frac{1}{2} \approx 0.5 \)
4. \( \tan 45^\circ = 1 \)
5. \( \cos 60^\circ = \frac{1}{2} \approx 0.5 \)
6. \( \sin 60^\circ = \frac{\sqrt{3}}{2} \approx 0.87 \)
7. \( \tan 30^\circ = \frac{\sqrt{3}}{3} \approx 0.58 \)
8. \( \cos 45^\circ = \frac{\sqrt{2}}{2} \approx 0.71 \)

Find \( x \). Round to the nearest hundredth if necessary.

9. \( x = 5.09 \)
10. \( x = 13.47 \)
11. \( x = 3.11 \)

Use a calculator to find the measure of \( \angle B \) to the nearest tenth.

6. \( \angle B \approx 70.5^\circ \)
7. \( \angle B \approx 36.9^\circ \)
8. \( \angle B \approx 49.2^\circ \)

9. GEOGRAPHY Diego used a theodolite to map a region of land for his class in geomorphology. To determine the elevation of a vertical rock formation, he measured the distance from the base of the formation to his position and the angle between the ground and the line of sight to the top of the formation. The distance was 43 meters and the angle was 36°. What is the height of the formation to the nearest meter?

31 m
Word Problem Practice

Trigonometry

1. **RADIO TOWERS** Kay is standing near a 200-foot-high radio tower.

![Diagram of Kay standing near a radio tower with a 49° angle and a 200-foot height.

200 ft

49°

Use the information in the figure to determine how far Kay is from the top of the tower. Express your answer as a trigonometric function.

\[ \sin 49° \]

3. **RAMP** A 60-foot ramp rises from the first floor to the second floor of a parking garage. The ramp makes a 15° angle with the ground.

60 ft

15°

How high above the first floor is the second floor? Express your answer as a trigonometric function.

\[ \sin 15° \]

4. **LINES** Jasmine draws line \( m \) on a coordinate plane.

What angle does \( m \) make with the \( x \)-axis? Round your answer to the nearest degree.

5. **NEIGHBORS** Amy, Barry, and Chris live on the same block. Chris lives up the street and around the corner from Amy, and Barry lives at the corner between Amy and Chris. The three homes are the vertices of a right triangle.

Barry

Chris

Amy

26°

64°

a. Give two trigonometric expressions for the ratio of Barry's distance from Amy to Chris' distance from Amy.

\[ \sin \theta \text{ or } \cos \theta \]

b. Give two trigonometric expressions for the ratio of Barry's distance from Chris to Amy's distance from Chris.

\[ \tan \theta \text{ or } \cot \theta \]

c. Give a trigonometric expression for the ratio of Amy's distance from Barry to Chris' distance from Barry.

\[ \tan 64° \]

Enrichment

Sine and Cosine of Angles

The following diagram can be used to obtain approximate values for the sine and cosine of angles from 0° to 90°. The radius of the circle is 1. So, the sine and cosine values can be read directly from the vertical and horizontal axes.

1. Use the diagram above to complete the chart of values.

<table>
<thead>
<tr>
<th>( x )°</th>
<th>0</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>40</th>
<th>50</th>
<th>60</th>
<th>70</th>
<th>80</th>
<th>90</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sin x )</td>
<td>0</td>
<td>0.17</td>
<td>0.34</td>
<td>0.5</td>
<td>0.64</td>
<td>0.77</td>
<td>0.87</td>
<td>0.94</td>
<td>0.98</td>
<td>1</td>
</tr>
<tr>
<td>( \cos x )</td>
<td>1</td>
<td>0.98</td>
<td>0.94</td>
<td>0.87</td>
<td>0.77</td>
<td>0.64</td>
<td>0.5</td>
<td>0.34</td>
<td>0.17</td>
<td>0</td>
</tr>
</tbody>
</table>

2. Compare the sine and cosine of two complementary angles (angles with a sum of 90°). What do you notice?

The sine of an angle is equal to the cosine of the complement of the angle.
**Angles of Elevation and Depression**

Many real-world problems that involve looking up to an object can be described in terms of an angle of elevation, which is the angle between an observer's line of sight and a horizontal line.

When an observer is looking down, the angle of depression is the angle between the observer's line of sight and a horizontal line.

**Example**

The angle of elevation from point A to the top of a cliff is 34°. If point A is 1000 feet from the base of the cliff, how high is the cliff?

Let \( x = \) the height of the cliff.

\[
1000\tan 34° = x
\]

Multiply each side by 1000.

\[
674.5 = x
\]

Use a calculator.

The height of the cliff is about 674.5 feet.

**Exercises**

1. **HILL TOP** The angle of elevation from point A to the top of a hill is 49°. If point A is 400 feet from the base of the hill, how high is the hill?
   - 460 ft

2. **SUN** Find the angle of elevation of the Sun when a 12.5-meter-tall telephone pole casts a 18-meter-long shadow.
   - 35°

3. **SKIING** A ski run is 1000 yards long with a vertical drop of 208 yards. Find the angle of depression from the top of the ski run to the bottom.
   - 12°

4. **AIR TRAFFIC** From the top of a 120-foot-high tower, an air traffic controller observes an airplane on the runway at an angle of depression of 19°. How far from the base of the tower is the airplane?
   - 348.5 ft
**NAME**

**DATE**  **PERIOD**

**Chapter 8**

**Skills Practice**

### 8-5 Angles of Elevation and Depression

Name the angle of depression or angle of elevation in each figure.

1. \(\angle FLS; \angle TLS\)

2. \(\angle RTW; \angle SWT\)

3. \(\angle DCB; \angle ABC\)

4. \(\angle WZP; \angle RPZ\)

**5. Mountain Biking**

On a mountain bike trip along the Gemini Bridges Trail in Moab, Utah, Nabuko stopped on the canyon floor to get a good view of the twin sandstone bridges. Nabuko is standing about 60 metres from the base of the canyon cliff, and the natural arch bridges are about 100 metres up the canyon wall. If her line of sight is 5 metres above the ground, what is the angle of elevation to the top of the bridges? Round to the nearest tenth degree.

about 57.7°

**6. Shadows**

Suppose the sun casts a shadow off a 35-foot building. If the angle of elevation to the sun is 60°, how long is the shadow to the nearest tenth of a foot?

about 20.2 ft

**7. Ballooning**

Angie sees a hot air balloon in the sky from her spot on the ground. The angle of elevation from Angie to the balloon is 40°. If she steps back 200 feet, the new angle of elevation is 10°. If Angie is 5.5 feet tall, how far off the ground is the hot air balloon?

about 50.1 ft

**8. Indirect Measurement**

Kyle is at the end of a pier 30 feet above the ocean. His eye level is 3 feet above the pier. He is using binoculars to watch a whale surface. If the angle of depression of the whale is 20°, how far is the whale from Kyle’s binoculars? Round to the nearest tenth foot.

about 96.5 ft

**Practice**

### 8-5 Angles of Elevation and Depression

Name the angle of depression or angle of elevation in each figure.

1. \(\angle TRZ; \angle YZR\)

2. \(\angle PRM; \angle LMR\)

3. **Water Towers**

A student can see a water tower from the closest point of the soccer field at San Lobos High School. The edge of the soccer field is about 110 feet from the water tower and the water tower stands at a height of 32.5 feet. What is the angle of elevation if the eye level of the student viewing the tower from the edge of the soccer field is 6 feet above the ground? Round to the nearest tenth.

about 13.5°

**4. Construction**

A roofer props a ladder against a wall so that the top of the ladder reaches a 30-foot roof that needs repair. If the angle of elevation from the bottom of the ladder to the roof is 55°, how far is the ladder from the base of the wall? Round your answer to the nearest foot.

about 21 ft

**5. Town Ordinances**

The town of Belmont restricts the height of flagpoles to 25 feet on any property. Lindsay wants to determine whether her school is in compliance with the regulation. Her eye level is 5.5 feet from the ground and she stands 36 feet from the flagpole. If the angle of elevation is about 25°, what is the height of the flagpole to the nearest tenth?

about 22.3 ft

**6. Geography**

Stephan is standing on the ground by a mesa in the Painted Desert. Stephan is 1.8 meters tall and sights the top of the mesa at 29°. Stephan steps back 100 meters and sights the top at 25°. How tall is the mesa?

about 296 m

**7. Indirect Measurement**

Mr. Dominguez is standing on a 40-foot ocean bluff near his home. He can see his two dogs on the beach below. If his line of sight is 6 feet above the ground and the angles of depression to his dogs are 34° and 48°, how far apart are the dogs to the nearest foot?

about 27 ft
34.98 m
49.95 m

1. What angle value needs to be maximized? Why?
2. Find the value of $x$ if $x = 10$ feet.
3. What is the value of $x$ if $x = 20$ feet?
4. What is the value of $x$ if $x = 25$ feet?
5. What is the value of $x$ if $x = 35$ feet?
6. What is the value of $x$ if $x = 50$ feet?
7. Which value of $x$ gives the greatest value of $a$? So, where is the best seat in the movie theater?
The Law of Sines and Law of Cosines

The Law of Sines

In any triangle, there is a special relationship between the angles of the triangle and the lengths of the sides opposite the angles.

\[ \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} \]

The Law of Cosines

Another relationship between the sides and angles of any triangle is called the Law of Cosines. You can use the Law of Cosines if you know three sides of a triangle or if you know two sides and the included angle of a triangle.

\[ a^2 = b^2 + c^2 - 2bc \cos A \]

Example 1

Find \( b \). Round to the nearest tenth.

\[
\begin{align*}
\sin C &= \frac{\sin B}{b} \\
\sin 45^\circ &= \frac{\sin 74^\circ}{b} \\
30 \sin 74^\circ &= 30 \sin 45^\circ \\
b &= 40.8 \\
\end{align*}
\]

Example 2

Find \( d \). Round to the nearest tenth.

\[
\begin{align*}
\sin B &= \frac{\sin E}{e} \\
\sin 58^\circ &= \frac{\sin 58^\circ}{24} \\
d &= 24 \sin 58^\circ \\
d &= 28.0 \\
\end{align*}
\]

Exercises

Find \( x \). Round to the nearest tenth.

1. 18.4
2. 26.2
3. 18.0
4. 7.7
5. 12.2
6. 30.4

Example 1

Find \( c \). Round to the nearest tenth.

\[
\begin{align*}
c^2 &= a^2 + b^2 - 2ab \cos C \\
c^2 &= 12^2 + 10^2 - 2 \cdot 12 \cdot 10 \cos 48^\circ \\
c^2 &= 144 + 100 - 240 \cos 48^\circ \\
c &= \sqrt{244 - 240 \cos 48^\circ} \\
c &\approx 28.0
\end{align*}
\]

Example 2

Find \( m \angle A \). Round to the nearest degree.

\[
\begin{align*}
a^2 &= b^2 + c^2 - 2bc \cos A \\
(82)^2 &= (80)^2 + (80)^2 - 2 \cdot 80 \cdot 80 \cos A \\
6400 &= 6400 - 3200 \cos A \\
\frac{1}{2} &= \cos A \\
60^\circ &= A
\end{align*}
\]

Exercises

Find \( x \). Round angle measures to the nearest degree and side measures to the nearest tenth.

1. 13.5
2. 11
3. 51
4. 29.8
5. 24.3
6. 53
**8-6 Skills Practice**

**The Law of Sines and Law of Cosines**

Find x. Round angle measures to the nearest degree and side lengths to the nearest tenth.

1. \[ \triangle ABC \]
   - \( \overline{AB} = 21.6 \) m
   - \( \overline{AC} = 7.3 \) m
   - \( \angle C = 48.8 \)°

2. \[ \triangle ABC \]
   - \( \overline{AB} = 16.8 \) m
   - \( \overline{AC} = 43.7 \) m
   - \( \angle C = 19.8 \)°

3. \[ \triangle ABC \]
   - \( \overline{AB} = 46.6 \) m
   - \( \overline{AC} = 40.1 \) m
   - \( \angle C = 88 \)°

4. \[ \triangle ABC \]
   - \( \overline{AB} = 52 \) m
   - \( \overline{AC} = 19.8 \) m
   - \( \angle C = 26.2 \)°

5. \[ \triangle ABC \]
   - \( \overline{AB} = 38.6 \) m
   - \( \overline{AC} = 67 \) m
   - \( \angle C = 11.9 \)°

16. Solve the triangle. Round angle measures to the nearest degree.

- \( m\angle A = 82 \)°
- \( m\angle B = 51 \)°
- \( m\angle C = 47 \)°

**8-6 Practice**

**The Law of Sines and Law of Cosines**

Find x. Round angle measures to the nearest degree and side lengths to the nearest tenth.

1. \[ \triangle ABC \]
   - \( \overline{AB} = 3.7 \) m
   - \( \overline{AC} = 166 \) m
   - \( \angle C = 14.2 \)°

2. \[ \triangle ABC \]
   - \( \overline{AB} = 12.9 \) m
   - \( \overline{AC} = 33.2 \) m
   - \( \angle C = 8.3 \)°

3. \[ \triangle ABC \]
   - \( \overline{AB} = 31.3 \) m
   - \( \overline{AC} = 16.9 \) m
   - \( \angle C = 16.6 \)°

4. \[ \triangle ABC \]
   - \( \overline{AB} = 48 \) m
   - \( \overline{AC} = 20.3 \) m
   - \( \angle C = 43 \)°

**INDIRECT MEASUREMENT** To find the distance from the edge of the lake to the tree on the island in the lake, Hannah set up a triangular configuration as shown in the diagram. The distance from location A to location B is 85 meters. The measures of the angles at A and B are 51° and 83°, respectively. What is the distance from the edge of the lake at B to the tree on the island at C? About 91.8 m
**8-6 Enrichment**

### Identities

An identity is an equation that is true for all values of the variable for which both sides are defined. One way to verify an identity is to use a right triangle and the definitions for trigonometric functions.

**Example 1**
Verify that \( \sin^2 A + \cos^2 A = 1 \) is an identity.

\[
\sin^2 A + \cos^2 A = 1
\]

To check whether an equation may be an identity, you can test several values. However, since you cannot test all values, you cannot be certain that the equation is an identity.

**Example 2**
Test \( \sin 2x = 2 \sin x \cos x \) to see if it could be an identity.

Try \( x = 20 \). Use a calculator to evaluate each expression.

- \( \sin 2x = \sin 40 \approx 0.643 \)
- \( 2 \sin x \cos x = 2 \sin(20) \cos(20) \approx 0.342 \times 0.940 \approx 0.324 \)

Since the left and right sides seem equal, the equation may be an identity.

### Exercises

Use triangle \( ABC \) shown above. Verify that each equation is an identity.

1. \( \frac{\cos A}{\sin A} = \frac{1}{\tan A} \)

2. \( \frac{\tan B}{\sin B} = \frac{1}{\cos B} \)

3. \( \tan B \cos B = \sin B \)

4. \( 1 - \cos^2 B = (\sin B)^2 \)

5. \( \cos 2x = \cos^2 x - \sin^2 x \)

6. \( \cos(90 - x) = \sin x \)

Yes; see students' work.

Yes; see students' work.
8-6 Graphing Calculator Activity
Solving Triangles Using the Law of Sines or Cosines
You can use a calculator to solve triangles using the Law of Sines or Cosines.

Example Solve \( \triangle ABC \) if \( a = 6 \), \( b = 2 \), and \( c = 7.5 \).

Use the Law of Cosines.

\[
\begin{align*}
a^2 &= b^2 + c^2 - 2bc \cos A \\
6^2 &= 2^2 + 7.5^2 - 2 \cdot 2 \cdot 7.5 \cos A
\end{align*}
\]

\[
m\angle A = \cos^{-1} \left( \frac{2^2 + 7.5^2 - 6^2}{2 \cdot 2 \cdot 7.5} \right)
\]

Use your calculator to find the measure of \( \angle A \).

Keystrokes: Enter: \( \cos^{-1} \) \( \left( \frac{2^2 + 7.5^2 - 6^2}{2 \cdot 2 \cdot 7.5} \right) \) \( \approx 36.06658826 \)

So \( m\angle A \approx 36 \). Use the Law of Sines and your calculator to find \( m\angle B \).

\[
\sin A = \frac{\sin B}{\frac{b}{\sin 36}}
\]

\[
m\angle B = \sin^{-1} \left( \frac{\sin 36 \cdot 2}{\sin 7.5} \right)
\]

Keystrokes: Enter: \( \sin^{-1} \) \( \left( \frac{\sin 36 \cdot 2}{\sin 7.5} \right) \) \( \approx 11.29896425 \)

So \( m\angle B \approx 11 \). By the Triangle Angle-Sum Theorem, \( m\angle C = 180 \) – (36 + 11) or 133.

Exercises Solve each \( \triangle ABC \). Round measures of sides to the nearest tenth and measures of angles to the nearest degree.

1. \( a = 9 \), \( b = 14 \), \( c = 12 \) \( m\angle A \approx 40 \), \( m\angle B \approx 89 \), \( m\angle C \approx 51 \)

2. \( m\angle C = 80 \), \( c = 9 \), \( m\angle A = 40 \) \( m\angle B = 60 \), \( a \approx 5.9 \), \( b \approx 7.9 \)

3. \( m\angle B = 45 \), \( m\angle C = 56 \), \( a = 2 \) \( m\angle A = 79 \), \( b \approx 1.4 \), \( c \approx 1.7 \)

4. \( a = 5.7 \), \( b = 6 \), \( c = 5 \) \( m\angle A = 62 \), \( m\angle B = 68 \), \( m\angle C = 50 \)

5. \( a = 11 \), \( b = 15 \), \( c = 21 \) \( m\angle A \approx 30 \), \( m\angle B \approx 43 \), \( m\angle C \approx 107 \)

8-7 Study Guide and Intervention
Vectors
Geometric Vector Operations A vector is a directed line segment representing a quantity that has both magnitude, or length, and direction. For example, the speed and direction of an airplane can be represented by a vector. In symbols, a vector is written as \( \vec{AB} \), where \( A \) is the initial point and \( B \) is the endpoint, or as \( \vec{v} \). The sum of two vectors is called the resultant. Subtracting a vector is equivalent to adding its opposite. The resultant of two vectors can be found using the parallelogram method or the triangle method.

Example Copy the vectors to find \( \vec{a} - \vec{b} \).

Method 1: Use the parallelogram method.

Copy \( \vec{a} \) and \( -\vec{b} \) with the same initial point. Complete the parallelogram. Draw the diagonal of the parallelogram from the initial point.

Method 2: Use the triangle method.

Copy \( \vec{a} \). Place the initial point of \( -\vec{b} \) at the terminal point of \( \vec{a} \). Draw the vector from the initial point of \( \vec{a} \) to the terminal point of \( -\vec{b} \).

Exercises Copy the vectors. Then find each sum or difference.

1. \( \vec{a} + \vec{b} \)

2. \( \vec{c} - \vec{d} \)

3. \( \vec{a} - \vec{b} \)

4. \( \vec{r} + \vec{t} \)

5. \( \vec{c} - \vec{d} \)
**Vehicles**

**Vectors**

**Vectors on the Coordinate Plane** A vector in standard position has its initial point at \((0, 0)\) and can be represented by the ordered pair \(B\). The vector at the right can be expressed as \(\vec{v} = \langle 5, 3 \rangle\).

You can use the Distance Formula to find the magnitude \(|\overline{AB}|\) of a vector. You can describe the direction of a vector by measuring the angle that the vector forms with the positive \(x\)-axis or with any other horizontal line.

**Example**

Find the magnitude and direction of \(\vec{a} = \langle 3, 5 \rangle\).

**Exercise**

Find the magnitude and direction of each vector.

1. \(\vec{a} = \langle -5, 2 \rangle\)
2. \(\vec{b} = \langle 0, 1 \rangle\)
3. \(\vec{c} = \langle 3, 4 \rangle\)
4. \(\vec{d} = \langle -2, 1 \rangle\)
5. \(\vec{e} = \langle -3, -4 \rangle\)
6. \(\vec{f} = \langle 4, 1 \rangle\)

Find each of the following for \(\vec{a} = \langle 2, 4 \rangle\), \(\vec{b} = \langle 3, -3 \rangle\), and \(\vec{c} = \langle 4, -1 \rangle\). Check your answers graphically.
8-7 Practice
Vectors
Use a ruler and a protractor to draw each vector. Include a scale on each diagram.
1. \( \mathbf{v} = 12 \) Newtons of force at 45° to the horizontal
2. \( \mathbf{w} = 15 \) miles per hour 70° east of north

Copy the vectors to find each sum or difference.
3. \( \mathbf{v} + \mathbf{r} \)
4. \( \mathbf{s} - \mathbf{t} \)

5. Write the component form of \( \mathbf{AB} \).

Find the magnitude and direction of each vector.
6. \( \mathbf{v} = (6, 11) \)
7. \( \mathbf{s} = (9, -7) \)

Find each of the following for \( \mathbf{a} = (-1, 5), \mathbf{b} = (7, 3), \) and \( \mathbf{c} = (1, -2) \). Check your answers graphically.
8. \( 2\mathbf{a} + \mathbf{b} \)
9. \( 2\mathbf{c} - \mathbf{b} \)

10. AVIATION A jet begins a flight along a path due north at 300 miles per hour. A wind is blowing due west at 30 miles per hour. Find (a) the resultant velocity of the plane. About 301.5 mph (b) the resultant direction of the plane. About 5.7° west of due north

8-7 Word Problem Practice
Vectors
1. WIND The vector \( \mathbf{v} \) represents the speed and direction that the wind is blowing. Suddenly the wind picks up and doubles its speed, but the direction does not change. Write an expression for a vector that describes the new wind velocity in terms of \( \mathbf{v} \).

2. SWIMMING Jan is swimming in a triathlon event. When the ocean water is still, her velocity can be represented by the vector \( \langle 2, 1 \rangle \) miles per hour. During the competition, there was a fierce current represented by the vector \( \langle -2, 4 \rangle \). What vector represents Jan’s velocity during the race? \( \langle 1, 0 \rangle \) mph

3. POLYGONS Draw a regular polygon around the origin. For each side of the polygon, associate a vector whose magnitude is the length of the corresponding side and whose direction points in the clockwise motion around the origin. What vector represents the sum of all these vectors? Explain. \( \langle 0, 0 \rangle \); Because the polygon forms a closed loop, the sum is the zero vector.

4. BASEBALL Rick is in the middle of a baseball game. His teammate throws him the ball, but throws it far in front of him. He has to run as fast as he can to catch it. As he runs, he knows that as soon as he catches it, he has to throw it as hard as he can to the teammate at home plate. He has no time to stop. In the figure, \( \mathbf{a} \) is the vector that represents the velocity of the ball after it is thrown. \( \mathbf{b} \) represents Rick’s velocity because he is running. Assume that Rick can throw just as hard when running as he can when standing still.

b. The angle between \( \mathbf{a} \) and \( \mathbf{b} \) is 90°. By running, did it help Rick get the ball to home plate faster than he would have normally been able to if he were standing still? Yes, it helped.
**Dot Product**

The dot product of two vectors represents how much the vectors point in the direction of each other. If \( \vec{v} \) is a vector represented by \( (a, b) \) and \( \vec{u} \) is a vector represented by \( (c, d) \), the formula to find the dot product is:

\[
\vec{v} \cdot \vec{u} = ac + bd
\]

Look at the following example:

Graph the vectors and find the dot product of \( \vec{v} \) and \( \vec{u} \) if \( \vec{v} = (3, -1) \) and \( \vec{u} = (2, 5) \).

\[
\vec{v} \cdot \vec{u} = (3)(2) + (-1)(5) = 6 - 5 = 1
\]

**Graph the vectors and find the dot products.**

1. \( \vec{v} = (2, 1) \) and \( \vec{u} = (-4, 2) \)
   - The dot product is \( -6 \)

2. \( \vec{v} = (3, -2) \) and \( \vec{u} = (1, 4) \)
   - The dot product is \( -5 \)

3. \( \vec{v} = (0, 3) \) and \( \vec{u} = (2, 4) \)
   - The dot product is \( 12 \)

4. \( \vec{v} = (-1, 4) \) and \( \vec{u} = (-4, 2) \)
   - The dot product is \( 12 \)

5. Notice the angle formed by the two vectors and the corresponding dot product. Is there any relationship between the type of angle between the two vectors and the sign of the dot product? Make a conjecture.

   **Yes, there is a relationship between the type of angle between the two vectors and the sign of the dot product. When the angle formed is acute, the sign of the dot product is positive and when the angle formed is obtuse, the sign of the dot product is negative.**
Chapter 8 Assessment Answer Key

Quiz 1 (Lessons 8-1 and 8-2)
Page 51
1. $8\sqrt{3}$
   $x = \sqrt{85}, \quad y = 2\sqrt{15}$
2. $\sqrt{17}, \quad y = \frac{81}{8}$
3. $\sqrt{137}$
4. acute

Quiz 2 (Lessons 8-3 and 8-4)
Page 51
1. $3\sqrt{2}$
2. $2\sqrt{3}$
3. 57.8
4. 14.6
5. $20\sqrt{3} + 20$ in.
6. 0.7880
7. 27
8. 3.3 ft
9. 27.1
10. 22.0

Quiz 3 (Lessons 8-5 and 8-6)
Page 52
1. $\angle QPR$
2. 3.9
3. $m\angle B = 104, \quad m\angle C = 27, \quad b = 23.2$
4. $m\angle R = 88, \quad m\angle T = 53, \quad m\angle S = 39$
5. $52^\circ$

Quiz 4 (Lesson 8-7)
Page 52
1. 24.8 units, 220°
2. 31.4 units, 158°
3. $\vec{c} + \vec{d}$
4. $-\vec{m}$
5. 6 mph, south

Mid-Chapter Test
Page 53

Part I

1. A

Part II

6. $x = 9, \quad y = 3\sqrt{3}$
   $x = 10\sqrt{3}, \quad y = 10\sqrt{6}$
7. $x = 12\sqrt{2}, \quad y = 24\sqrt{2}$
8. right
9. 54
### Vocabulary Test Form 1

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<td>2. Pythagorean triple</td>
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<td>3. false, trigonometric ratio</td>
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<tr>
<td>4. true</td>
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<td></td>
</tr>
<tr>
<td>5. angle of depression</td>
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<td>6. trigonometry</td>
<td>16.</td>
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<td>7. sine</td>
<td>17.</td>
<td></td>
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<tr>
<td>8. tangent</td>
<td>18.</td>
<td></td>
</tr>
<tr>
<td>A method to describe a vector using its horizontal and vertical change from its initial to terminal point.</td>
<td>19.</td>
<td></td>
</tr>
<tr>
<td>In a right triangle, the sum of the squares of the measures of the legs equals the square of the measure of the hypotenuse.</td>
<td>B:</td>
<td>69</td>
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**Chapter 8 Assessment Answer Key**
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<tr>
<td>5.</td>
<td>D</td>
<td>D</td>
</tr>
<tr>
<td>6.</td>
<td>J</td>
<td>F</td>
</tr>
<tr>
<td>7.</td>
<td>A</td>
<td>A</td>
</tr>
<tr>
<td>8.</td>
<td>G</td>
<td>G</td>
</tr>
<tr>
<td>9.</td>
<td>B</td>
<td>H</td>
</tr>
<tr>
<td>10.</td>
<td>H</td>
<td>H</td>
</tr>
</tbody>
</table>

B: 56.4 ft  B: 73.2 ft
Chapter 8 Assessment Answer Key

Form 2C
Page 61

1. \( \sqrt{10} \)

2. 13

3. \( 2\sqrt{7} \)

4. \( \sqrt{4000} \) or \( 20\sqrt{10} \)

5. \( \sqrt{300} \) or \( 10\sqrt{3} \)

6. \( 11\sqrt{2} \)

7. \( 2\sqrt{3} \)

8. \( x = 6, y = 12 \)

9. 9.7

10. 69

11. 68

Page 62

12. 267.9 m

13. 11°

14. 43.2

15. 14.3

16. 15.6 ft

17. 20

18. 33.0

19. \( \sqrt{170} \approx 13 \) units; 237.5°

20. 1

B: 1
Chapter 8 Assessment Answer Key

Form 2D
Page 63

1. \(3\sqrt{10}\)

2. \(26\)

3. \(\sqrt{39}\)

4. \(\sqrt{7300} \text{ or } 10\sqrt{73}\)

5. \(\sqrt{432} \text{ or } 12\sqrt{3}\)

6. \(15\sqrt{2}\)

7. \(7\sqrt{3}\)

8. \(x = 12, y = 24\)

9. \(8.6\)

10. \(70°\)

11. \(67°\)

Page 64

12. \(352.7 \text{ m}\)

13. \(9°\)

14. \(32.3\)

15. \(6.2°\)

16. \(15.2 \text{ ft}\)

17. \(19\)

18. \(38.2\)

19. \(\sqrt{109} \approx 10.4 \text{ units, } 253.3°\)

20. \(6 \text{ or } 10\)

B:
Chapter 8 Assessment Answer Key

Form 3  
Page 65

1. \( \frac{\sqrt{6}}{9} \)
2. 2
3. 3
4. \( 2\sqrt{10} \)
5. \( \frac{12}{5} \)
6. 12.5
7. 250 km
8. right
9. \( 12\sqrt{6} \)

10. \( 24 + 8\sqrt{3} \)

\[ x = 5\sqrt{3}, \quad y = 10\sqrt{3} \]

11. \[ (-4 + 8\sqrt{3}, -2) \]

or \[ (-4 - 8\sqrt{3}, -2) \]

Page 66

13. \( \frac{5\sqrt{6}}{3}, \quad CD = 5 \frac{\sqrt{6}}{3} + 5\sqrt{2} + 12 \)

14. 147 ft

15. 2 ft

16. 37.4

17. 253.7 yd

18. 37° or 143°

19. 31.4 units, 261°

20. B: 71.1 ft and 84.5 ft
## Chapter 8 Assessment Answer Key

### Extended-Response Test, Page 67

### Scoring Rubric

<table>
<thead>
<tr>
<th>Score</th>
<th>General Description</th>
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</thead>
</table>
| 4     | Superior            | • Shows thorough understanding of the concepts of geometric mean, special right triangles, altitude to the hypotenuse theorems, Pythagorean Theorem, solving triangles, SOH CAH TOA, Law of Sines, and Law of Cosines.  
         |                     | • Uses appropriate strategies to solve problems.  
         |                     | • Computations are correct.  
         |                     | • Written explanations are exemplary.  
         |                     | • Figures are accurate and appropriate.  
         |                     | • Goes beyond requirements of some or all problems. |
| 3     | Satisfactory        | • Shows an understanding of the concepts of geometric mean, special right triangles, altitude to the hypotenuse theorems, Pythagorean Theorem, solving triangles, SOH CAH TOA, Law of Sines, and Law of Cosines.  
         |                     | • Uses appropriate strategies to solve problems.  
         |                     | • Computations are mostly correct.  
         |                     | • Written explanations are effective.  
         |                     | • Figures are mostly accurate and appropriate.  
         |                     | • Satisfies all requirements of problems. |
| 2     | Nearly Satisfactory | • Shows an understanding of most of the concepts of geometric mean, special right triangles, altitude to the hypotenuse theorems, Pythagorean Theorem, solving triangles, SOH CAH TOA, Law of Sines and Law of Cosines.  
         |                     | • May not use appropriate strategies to solve problems.  
         |                     | • Computations are mostly correct.  
         |                     | • Written explanations are satisfactory.  
         |                     | • Figures are mostly accurate.  
         |                     | • Satisfies the requirements of most of the problems. |
| 1     | Nearly Unsatisfactory | Final computation is correct.  
         |                     | • No written explanations or work is shown to substantiate the final computation.  
         |                     | • Figures may be accurate but lack detail or explanation.  
         |                     | • Satisfies minimal requirements of some of the problems. |
| 0     | Unsatisfactory      | • Shows little or no understanding of most of the concepts of geometric mean, special right triangles, altitude to the hypotenuse theorems, Pythagorean Theorem, solving triangles, SOH CAH TOA, Law of Sines and Law of Cosines.  
         |                     | • Does not use appropriate strategies to solve problems.  
         |                     | • Computations are incorrect.  
         |                     | • Written explanations are unsatisfactory.  
         |                     | • Figures are inaccurate or inappropriate.  
         |                     | • Does not satisfy requirements of problems.  
         |                     | • No answer given. |
Chapter 8 Assessment Answer Key
Extended-Response Test, Page 67
Sample Answers

In addition to the scoring rubric found on page A31, the following sample answers may be used as guidance in evaluating open-ended assessment items.

1. \[
\frac{10}{6} = \frac{6}{x}
\]
\[10x = 36\]
\[x = \frac{18}{5}\]

2a. No, his work is not correct. The triangle is not a right triangle so he cannot use the altitude to the hypotenuse theorem. Instead he must use the 30°-60°-90° triangle relationships and obtain
\[x = 2\sqrt{3}\].

b. Using the 30°-60°-90° triangle relationships, \(RQ = 4\), and therefore \(PQ\) would have to equal 8, so \(PS = 8 - 2 = 6\).

c. No, the sides are not in a ratio of 1:1:√2.

3. If you were finding the length of a side \(x\) you would use \(\sin\) of the given angle. If you are finding an angle \(x\) you would use the \(\sin^{-1}\) of the ratio of two sides to find the angle. For example, if \(\sin x = \frac{3}{4}\), use \(\sin^{-1}\frac{3}{4}\) to find \(x\). If \(\sin 20° = \frac{x}{5}\) find the sine of 20° and multiply by 5 to find \(x\), the length of the unknown side.

4. Student should draw a right triangle and label the angle of elevation up from the horizontal and the angle of depression down from the horizontal. The angle of elevation has the same measure as the angle of depression.

5. Let \(\angle B = 110\), \(\angle A = 20\), and \(b = 150\). Use the Law of Sines to find the missing lengths. The length of \(a\) is found using \[
\frac{\sin 110°}{150} = \frac{\sin 20°}{a}
\]. The third angle is found by evaluating \(180 - (\angle B + \angle A)\). In this problem, \(\angle C = 50\). The length of \(c\) is found by using \[
\frac{\sin 110°}{150} = \frac{\sin 50°}{c}
\].

In this problem, \(c \approx 122.3\).

6. No, her plan is not a good one. Irina should use the Law of Cosines to find angle \(B\). Then she can use the Law of Sines to find either angle \(A\) or angle \(C\) and subtract the two angles she finds from 180 degrees to get the measure of the third angle.
Chapter 8 Assessment Answer Key

Standardized Test Practice
Page 68

1. • ○ ○ ○

10. ○ ○ ○ ○

11. ○ • ○ ○

2. • ○ ○ ○

12. ○ ○ ○ ○

3. ○ ○ ○ ○

13. ○ ○ ○ ○

4. ○ ○ ○ ○

14. ○ ○ ○ ○

5. ○ ○ ○ ○

15.

6. ○ ● ○ ○

7. ○ ○ ○ ○

8. ○ ● ○ ○

9. ○ ○ ○ ○

16.
Chapter 8 Assessment Answer Key

Standardized Practice Test

17. **Def. of ≅ segments**

18. \( \overline{HJ} + \overline{JK} = \overline{HK}; \overline{KL} + \overline{LM} = \overline{KM} \)

19. \( m\angle 1 = 32, m\angle 2 = 82, m\angle 3 = 66, m\angle 4 = 114, m\angle 5 = 57 \)

20. **7**

21. **19**

22. \( \angle 3, \angle 4, \angle 5, \angle 7, \angle ABC \)

yes, both are 30°-60°-90° triangles; AA Similarity

23a. **36**

23b. 2:1