The psychiatrist and the decision analyst both earn their living by getting people's heads straightened out.

Z. Gozinto
Celebrated Italian Mathematician*

The case method of instruction calls for discussion of a situation faced by a decision maker and points out how to make a good decision under the condition described. Often, however, actual situations are too difficult, confusing, and contain many distracting and extraneous conditions. Therefore, case methods are usually simplified descriptions of real life situations. However, it is also true that a fictional situation presenting a mixture of reality and imagination offers an excellent way to obtain insight into the decision analysis of real life conditions.

The case discussed below represents such a situation. The plight of the decision maker has been analyzed many times, and in fact this case has become a *cas célèbre*. However, to my knowledge this is the first time that the case has been analyzed by a sophisticated decision analyst steeped in the modern theory of rational human behavior. Certainly, if anyone should have had his head straightened out, it was Hamlet, the Prince of Denmark.

**A CASE STUDY OF A DECISION ANALYSIS: HAMLET'S SOLILOQUY**

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One of the difficulties in retrospective studies of actual decisions is the veiled nature of the human mind. Whatever documents or recordings of historical decisions

DECISION ANALYSIS

*Column Editor’s Note: Back in the prehistoric days of Operations Research and Management Science, I was delivering a lecture on the parts requirements problem of production control. One of the diagrams associated with the problem had come to be known as the “Gozinto” diagram. One member of the audience, Dr. George B. Dantzig, today famous as the discoverer of the Simplex Method, queried me about the mathematician Gozinto. Astounded that such a well-educated mathematician did not know about Gozinto, I said “Why, this theorem was discovered by the celebrated Italian, Z. Gozinto.” Dr. Dantzig thereupon asked what the Z stood for, and I replied “Zepartzat, of course!” From such an humble origin arose the internationally renowned reputation of Z. Gozinto.
exist, the question of what was actually in the minds of those responsible for the decision at the sticking point is at best hypothetical.

There is one famous decision which has been recorded in terms of the stream of consciousness of the decision maker: Hamlet’s ‘‘To be or not to be’’ soliloquy. (This particular decision is fiction, but, as the Freudians have pointed out, the most perceptive characterization of an ethos stems from literature rather than from scholarship.) Shakespeare presents us with a vivid recital of the thought processes of a man making up his mind about a critical step in his life. It is revealing to examine this case in light of modern understanding of decision analysis.

Hamlet starts off with an exemplary first stage of his decision tree:

FIGURE 1.

A

TO BE

B

NOT TO BE

Some decision theorists might question whether Hamlet had identified all possible actions available to him. That question misses the extraordinary power and generality of Hamlet’s representation of his options. In essence, he took the set of worlds available to him and boldly divided them into two exclusive and exhaustive classes. The secondary question of how he might implement either of these alternatives — e.g., ‘‘with a bare bodkin’’ for B, or by some other method — clearly could be left until he had decided which of these two classes of worlds he preferred to be in [Dalkey, 1979].

Hamlet then exemplifies another principle only recently formalized in decision theory; namely, he proceeds to evaluate his initial step to see if there is a clear-cut resolution of his problem at this stage. He formulates his multidimensional utility function for the two outcomes. ‘‘Whether ’tis nobler in the mind to suffer the slings and arrows of outrageous fortune . . . .’’ There are two basic dimensions, nobility and suffering. Hamlet does not, at this stage, determine precise trade-offs for these two. In fact, as he recognizes, the two may not be independent; a certain amount of nobility might be achieved by enduring the suffering. Thus, he leaves the precise structure of his utility function until a possible later stage when this aspect might be needed. Denoting the nobility dimension by N and the suffering dimension by S, it is clear that outcome A is positive on N and highly negative on S.

He then turns to alternative B and has a little trouble for a moment. ‘‘Or else take arms against a sea of troubles, and by opposing, end them.’’ Clearly, B offers 0 on S (i.e., no more suffering), but what about N? What occurs at this point is not completely unambiguous. Either Hamlet determines tacitly that S is overwhelmingly more important than N, or else he estimates that the dependency between N and S is such that a 0 value for S indicates the same for N. At any event, he goes on to state a tentative resolution to his problem; B is preferable to A. ‘‘To die — to sleep — no more; and by a sleep to say we end the heartache and the thousand natural shocks that flesh is heir to.’’ S is thus the critical dimension in Hamlet’s utility function.

Hamlet, as befits a rational decision maker, does not stop with his preliminary
analysis. He realizes there may be some gain in exploring the decision tree further, and he starts with $B$:

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FIGURE 2.

\[
\begin{align*}
B & \quad \text{NO DREAM} \\
& \quad p \quad \text{DREAM} \\
& \quad 1-p \\
\end{align*}
\]
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"To die, to sleep, perchance to dream." There is a nonzero probability $1-p$ of dreaming, and Hamlet perceives that this expansion may upset his tentative selection of $B$. "Ay, there's the rub." He quickly expands the dream node further, where $q_i$ indicates the probability of dream$_i$:

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FIGURE 3.

\[
\begin{align*}
\text{DREAM} & \quad q_1 \quad \text{DREAM}_1 \\
& \quad \Sigma q_i \quad \text{DREAM}_2 \\
1 \quad - \Sigma q_i \\
1 & \quad \text{DREAM}_n \\
\end{align*}
\]
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"For in that sleep of death what dreams may come . . . ." Some of the possible dreams might not be so bad, but apparently some are sheer horrors. The probabilities are unknown, but whatever they might be, Hamlet perceives that his original tentative decision was too hasty — "... must give us pause."

At this point Shakespeare leaves the actual decision as an exercise for the reader. I have worked the exercise, and the reader will forgive me for sparing him the bother. Hamlet recognizes that his problem is a decision with incomplete information — he does not know the probabilities $p$ and $q_i$. Although he has some uncertainties (brought out later) about the "game against nature" approach to decisions with incomplete information, he nevertheless adopts it.

We can express Hamlet's decision problem in terms of a payoff matrix.

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TABLE 1.

<table>
<thead>
<tr>
<th></th>
<th>No dream</th>
<th>Dream$_1$</th>
<th>Dream$_2$</th>
<th>Dream$_n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>-10</td>
<td>-10</td>
<td>-10</td>
<td>-10</td>
</tr>
<tr>
<td>B</td>
<td>0</td>
<td>100</td>
<td>-1000</td>
<td>-5</td>
</tr>
</tbody>
</table>
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The numbers are, of course, only illustrative. Negative 10 represents "slings and arrows of outrageous fortune, a sea of troubles," etc. This is his payoff if he chooses $A$, whatever the afterlife is like. Zero indicates "To sleep, no more . . . ." The 100
and $-5$ indicate various dreams, some possibly pleasant. The $-1000$ represents what Hamlet later in the soliloquy calls "something dreadful."

Since he does not know the probabilities of the afterlife states, he makes the assumption that the payoff matrix is for a game where an iminical being is in charge of the afterlife and will pick the state that will trouble him most. He, of course, then wants to pick the action best for him, taking into account the proclivities of his antagonist. The solution to this game is the well-known max-min strategy; he selects the action which (in this case where the significant payoffs are negative) minimizes the worst his opponent can do to him. For action $A$ the worst is $-10$, independently of the other’s action. For action $B$ the worst is $-1000$. Thus, the max-min strategy is $A$, where he will do no worse than $-10$.

Hamlet is rather in luck with this matrix; it has a solution in pure strategies. If it had been otherwise, as we now know, the solution would be a mixed strategy, i.e., to flip some lopsided coin to decide between $A$ and $B$. (In that case Shakespeare would have had to write at least two plays, depending on the outcome of Hamlet’s coin flip, and the subsequent course of English drama would have looked entirely different.) Subsequent events in the play show that selecting the first alternative was indeed Hamlet’s decision.

If this were the totality of the case study, it would be in itself a rather remarkable anticipation of recent developments in decision theory. But Hamlet then goes on to an ultramodern step — assessment of his own decision analysis; in effect, he carries out a post-mortem examination of his decision. This step is associated with a topic of growing interest, namely, the value of analysis. For Hamlet, the $64$ question is whether analysis is the way to go at all; as the soliloquy proceeds, he clearly flirts with the notion that analysis ("the pale cast of thought") may be counterproductive.

He first establishes in his own mind that his procedure is common practice. "There’s the respect that makes calamity of so long life." If most people didn’t follow the same line of analysis as he had gone through, they would be dead. He amplifies his utility function on the $S$ dimension "... whips and scorns of time, th’oppressor’s wrong, the proud man’s contumely, the pangs of dispriz’d love, the law’s delay, the insolence of office, the spurns that patient merit of th’unworthy takes, fardels, grunting and sweating under a weary life." In contrast to this, on alternative $B$ there is "something dreadful after death." He concludes that men generally opt for the sure, if highly negative, $S$, then the less sure unknown threat. "Makes us rather bear the ills we have than fly to others that we know not of?"

After reviewing his analysis and deciding that it is general practice, he concludes that it is rather poor. "Thus conscience does make cowards of us all; and thus the native hue of resolution is sicklied o’er with the pale cast of thought; and enterprises of great pitch and moment, with this regard, their current turn away, and lose the name of action.” As far as Hamlet is concerned, the max-min solution to decisions under uncertainty is a dubious way to select an action.

It is not clear whether Hamlet at this point would have started over and tried a Bayes solution to his problem, i.e., attempt an estimate of the relevant probabilities — such as assuming uniform probabilities $p = \frac{1}{2}$ for dream and not-dream, and uniform probabilities $q_i = \frac{1}{n}$ for the various potential dreams — then, using these, compute the expected value of the actions of $A$ and $B$. He was interrupted by the fair Ophelia, and his decision to be led to events that he had not dreamt of, not having expanded node $A$.
Decision-theoretic terminology was still in a formative stage in Shakespeare’s time, thus Hamlet’s remark to his friend “There are more things in heaven and earth, Horatio, than are dreamt of in your philosophy,” would probably be expressed in modern terminology as “There are states of nature, Horatio, not included in your frame of reference.”

REFERENCE

EPILOGUE BY THE COLUMN EDITOR

As Professor Dalkey points out, Hamlet tries to assess his own method of decision analysis; in fact, he even questions the value of taking action in his absurd and corrupt world. But modern decision theory considers no-action as simply a “null-action,” and recognizes that no-action is just one possible action taken by default. Therefore, no-action must be evaluated as any other action in terms of subjective expected utility. This is apparently overlooked by Hamlet, and is often overlooked even today.

Hamlet’s utility function includes only benefits to himself, and he appears to disregard all others. Does this anticipate Adam Smith and express the belief that an invisible hand leads to the best of all possible worlds?

Now to the most serious trap, often ignored, which is that of confusing action with outcome. As decision analysts know so well, good action can lead to bad outcome; bad action can lead to good outcome. When Hamlet talks about not to be, this decision leads with a high probability to the outcome not to be, although suicide attempts often fail. The action to be is really not a decision but an uncontrollable outcome. In fact, Hamlet makes the decision to be but ends up not to be because Laertes does kill Hamlet. Alternatively, this could be put into the framework of planning horizons. When Hamlet decides to be, it leads to a short-range outcome of to be, and so his decision avoids for a short period only the penalty involved had he decided not to be.

It is recognized that prescriptions of the theory of rational behavior are often not followed by decision makers; to put it bluntly, people often do not want to have their heads straightened out. This is of course no surprise to either decision analysts or psychiatrists.

A.Z.