If \( y = x^2 + 1 \), what is the smallest positive value of \( x \) such that \( \sin y \) is a relative maximum?

(A) 0.756   (B) 0.841   (C) 1
(D) 1.463   (E) 1.927
If \( f(1) = 2 \) and \( f'(1) = 5 \), use the equation of the line tangent to the graph of \( f \) at \( x = 1 \) to approximate \( f(1.2) \).

(A) 1    (B) 1.2    (C) 3    (D) 5.4    (E) 9
What is the approximate % change in the volume of a sphere when the radius changes from 10 to 10.1 centimeters?

(A) 10%   (B) 1 %   (C) 3%   (D) 3.01%   (E) 1.333%
The sides of right triangle ABC with hypothenuse AC = 13 cm. are changing. Leg AB is decreasing at a rate of .5 cm/sec.

(a) How fast is the other leg changing when AB = 5?

(b) How fast is the area changing at this time?

(c) How fast is angle A changing at this time?
In a right triangle with fixed hypotenuse of 13 what are the lengths of the legs at the moment when the rate that one leg is increasing is twice the rate at which the other leg is decreasing?
Find the dimensions and the area of the rectangle with maximum area that can be inscribed so that two vertices are on the x-axis and the other two are on the graph of $y = \sqrt{(9-x^2)}$. (Calculator active). Show all work.
Given: \( f(x) = -\frac{x^4}{4} + 2x^3 + 8x^2 \)

(a) Find intervals where \( f(x) \) is increasing.
(b) Find intervals where \( f(x) \) is concave up.
(c) State the extreme values of \( f(x) \). Include the type of extrema and the \( x \)-value where each occurs.
Given the following information, sketch a graph of \( y = f(x) \).

a) Domain: \(-3 \leq x \leq 5\)
b) \( f(1) = 0 \)
c) \( f' > 0 \) when \(-1 < x < 3\) or \( 3 < x \); \( f' = 0 \) when \( x = -1, 3 \); \( f' < 0 \) otherwise
d) \( f'' > 0 \) when \( x < 1 \) or \( x > 3 \); negative otherwise
1. A
\[ \sin y = M \]
M has a max when \( M' \) changes from + to -

\[ M' = \cos y \cdot y' = \cos (x^2 + 1) \cdot 2x \]

Analytically:
0 = \( \cos (x^2 + 1) \) or 0 = 2x

\[ x^2 + 1 = \frac{\pi}{2} \text{ or } 3\frac{\pi}{2} \ldots \text{ OR } x = 0 \]

Since \( x \) must be positive but not too big (look at choices)
\[ x^2 + 1 = \frac{\pi}{2} \]
\[ x = \sqrt{\frac{\pi}{2} - 1} \approx 0.7... \]

2. C
Linearization/tangent: \( y = 5(x-1) + 2 \) so \( y(1.2) = 5(0.2) + 2 - 3 \)

3. C
\[ dV = 4\pi rdr = \frac{3dr}{r} = \frac{3(1)}{10} = 3\% \]

Answers:

1. A
2. C
3. C
\[
\frac{dx}{dt} = -\frac{1}{2} \text{ cm/sec}
\]

\[
x^2 + y^2 = 13^2 \Rightarrow 2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 0
\]

\[
x \frac{dx}{dt} + y \frac{dy}{dt} = 0
\]

When \(x = 5, y = 12 \Rightarrow 5(-\frac{1}{2}) + 12 \left(\frac{dy}{dt}\right) = 0
\]

\[
\Rightarrow \frac{dy}{dt} = -\frac{5}{24} \text{ cm/sec.}
\]

\[
A = \frac{1}{2}xy \Rightarrow \frac{dA}{dt} = \frac{1}{2} \left( x \frac{dy}{dt} + y \frac{dx}{dt} \right)
\]

\[
= \frac{1}{2} \left( 5 \left(\frac{5}{24}\right) + 12 \left(-\frac{1}{2}\right) \right)
\]

\[
= -\frac{1}{2} \left( -\frac{25}{24} - 6 \right)
\]

\[
= -\frac{119}{48} \text{ cm}^2/\text{sec}
\]

\[
\cos \theta = \frac{y}{13} \Rightarrow -\sin \theta \frac{d\theta}{dt} = \frac{1}{13} \frac{dx}{dt}
\]

\[
-\frac{12}{13} \frac{d\theta}{dt} = \frac{1}{13} \left(-\frac{1}{2}\right)
\]

\[
\frac{d\theta}{dt} = \frac{1}{24} \text{ rad/sec.}
\]
\[-\frac{dy}{dt} = 2 \frac{dx}{dc} \quad \Rightarrow \quad x^2 + y^2 = 13^2\]

\[\frac{dx}{dt} x + \frac{dy}{dt} y = 0\]

\[x \frac{dx}{dt} + \sqrt{169-x^2} = 2 \frac{dx}{dc} = 0\]

\[x = 2\sqrt{169-x^2}\]

\[x^2 = 4(169-x^2)\]

\[5x^2 = 169(4)\]

\[x^2 = \frac{169(4)}{5} \Rightarrow y^2 = 169 - 169(4)\]

\[x = \frac{26}{\sqrt{5}} = \frac{169}{5}\]

\[y = \frac{13}{\sqrt{5}}\]
\( y = \sqrt{9 - x^2} \)

\[ A = 2xy = 2x \sqrt{9 - x^2} \quad \text{if} \quad 0 \leq x \leq 3 \]

\[ A' = 2 \left( \sqrt{9 - x^2} \right) + 2x \left( \frac{-2x}{2 \sqrt{9 - x^2}} \right) \]

\[ A' = 0 = \sqrt{9 - x^2} - \frac{x^2}{\sqrt{9 - x^2}} = \frac{9 - 2x^2}{\sqrt{9 - x^2}} \]

\[ x = \frac{3}{\sqrt{2}} \quad \text{(no - \frac{3}{\sqrt{2}} due to domain)} \]

\text{Compare endpoints: } A(0) = A(3) = 0 \quad \text{min area.}

\text{Critical pt: } A \left( \frac{3}{\sqrt{2}} \right) = 2 \left( \frac{3}{\sqrt{2}} \right) \left( \sqrt{9 - \frac{9}{2}} \right) = 2 \left( \frac{3}{\sqrt{2}} \right) \left( \frac{\sqrt{9}}{2} \right)

= 9 \quad \text{max area.}
Given: \( f(x) = -\frac{x^4}{4} + 2x^3 + 8x^2 \)

\[
f'(x) = -x^3 + 6x^2 + 16x = 0
\]

\[-x(x^2 - 6x - 16) = 0
\]

\[-x(x-8)(x+2) = 0
\]

\[f \text{ inc when } f' > 0 \implies x \leq 2, 0 < x < 8
\]

\[f''(x) = -3x^2 + 12x + 16 = 0
\]

\[x = \frac{-12 \pm \sqrt{444 + 192}}{-6}
\]

\[x = \frac{-12 \pm \sqrt{636}}{-6}
\]

\[x = \frac{2 \pm 4\sqrt{31}}{6}
\]

\[x = \frac{2 \pm 2\sqrt{31}}{3}
\]

\[f \text{ concave up when } f'' > 0 \implies 2 \frac{-2\sqrt{31}}{3} < x < 2 + \frac{2\sqrt{31}}{3}
\]

\[f \text{ has max when } f' \text{chg from + to -} \implies x = -2, 8
\]

\[f(-2) = -\frac{1}{4}(-2)^4 + 2(-2)^3 + 8(-2)^2
\]

\[= -4 + 16 + 64 = 76
\]

\[f(8) = -\frac{1}{4}(8)^4 + 2(8)^3 + 8(8)^2
\]

\[= 8^3 \left[\frac{1}{4}(8) + 2 + 1\right]
\]

\[= 8^3 \left[\frac{2}{4} + 2 + 1\right]
\]

\[= 8^3 = 512
\]

Abs. max 512 at x = 8

Local \( \text{ max } 76 \) at x = -2

Local \( \text{ min } 0 \) at x = 0

(No abs. min as fn 4th deg, u neg. lead coeff)
Given the following information, sketch a graph of $y = f(x)$.

a) Domain: $-3 \leq x \leq 5$

b) $f(1) = 0$

c) $f' > 0$ when $-1 < x < 3$, $3 < x$; $f' = 0$ when $x = -1, 3$; $f' < 0$ otherwise

d) $f'' > 0$ when $x < 1$ or $x > 3$;